Mawhin, Jean (ed.); **Ponce, Augusto (ed.)**
[De La Vallée Poussin, Charles-Jean]
Mathematical analysis course by Charles-Jean de La Vallée Poussin. Mathematical reconstruction of war damage. (Cours d'analyse de Charles-Jean de La Vallée Poussin. Reconstruction mathématique d'un dommage de guerre.) (French) Zbl 08196031
Mémoires de la Classe des Sciences. Collection in 8, 4e Série 14. Bruxelles: Académie Royale de Belgique (ISBN 978-2-8031-1036-0/print; 978-2-8031-1037-7/ebook). lvii, 494 p. (2025).

Within the English-speaking mathematical community, as well as amongst those with a natural affinity for the language of Shakespeare, Walter Rudin's (1921–2010) textbooks *Principles of Mathematical Analysis* and *Real and Complex Analysis* have for several decades been regarded as classics for anyone studying mathematical analysis. Their influence is due not only to the material they present, which has by now largely become canonical, but also to the particularly concise, rigorous, and well-structured manner in which that material is organized.

It is perhaps less widely known today that a comparable role was played in the French-speaking mathematical world as well as in those parts of the world where Molière's language exerts a certain influence, during the first half of the twentieth century by the *Cours d'Analyse infinitésimale* of the Belgian mathematician Charles-Jean de La Vallée Poussin (1866–1962). The comparison with Rudin is not entirely accidental. De La Vallée Poussin was himself one of the leading analysts of his time: independently of Jacques Hadamard, he proved the prime number theorem, and he made important contributions to integration theory, Fourier series, approximation theory, conformal mappings, and potential theory. His *Cours*, published in two volumes, enjoyed remarkable longevity and international influence. The first volume, initially published in 1903, went through twelve editions, the last appearing in 1959, whereas the second, first published in 1906, went through nine editions, the last in 1957.

This longevity cannot be explained solely by the reputation of its author. The *Cours* is an intrinsically remarkable work. More than a century after it was written, its terminology, some of its methods, and even the organization of several topics inevitably bear the marks of their time. Nevertheless, it also displays exceptional clarity of exposition, a constant concern for rigor, and considerable pedagogical skill. Above all, de La Vallée Poussin possessed a remarkable ability to assimilate a vast body of mathematics that was still new, rapidly developing, and far from having reached its final form, to identify its central ideas, and to reorganize them into a coherent exposition for a mathematically mature audience. The very design of the *Cours*, which distinguished between a "large text" intended for the main course and a "small text" containing the developments needed for more advanced study, reflects this ambition: to make the ideas accessible without ever sacrificing rigor.

The innovative character of the treatise is especially evident in its treatment of integration theory. The second edition of Volume I, published in 1909, was the first textbook to introduce the then very recent theory of the Lebesgue integral, developed only a few years earlier. De La Vallée Poussin did not merely reproduce recent results: he reworked proofs, adapted them to the requirements of teaching, and progressively incorporated results from his own research. The second and third editions of the *Cours* thus became, for many mathematicians in the early twentieth century, an important route into Lebesgue integration and its applications, particularly to Fourier series.

It is precisely the third edition of Volume II that lies at the center of the present book, edited by Jean Mawhin and Augusto Ponce, both professors of mathematics at the Université catholique de Louvain. Its history is unusual. In August 1914, during the sack of Louvain, German troops set fire to, among other buildings, historic buildings of the University, completely destroying its library. The Uystpruyst printing house, located nearby and then preparing the third edition of Volume II, was also destroyed, together with the material intended for that edition. This edition, which was to contain a substantially expanded treatment of set theory and the Lebesgue integral, never appeared.

The fourth edition did not reconstruct the lost text. Material difficulties after the war led de La Vallée Poussin to return largely to content closer to that of the earlier editions, while much of the new material on Lebesgue integration was published elsewhere. Subsequent editions essentially retained this revised

organization.

The possibility of reconstructing the lost edition arose much later, in the course of work devoted to the publication of de La Vallée Poussin's collected works. One of his great-grandsons had preserved a collection of printer's proofs. Among them were unpublished portions of the third edition of Volume II: the end of the chapter on improper integrals and exact total differentials, the whole chapter on multiple Riemann and Lebesgue integrals, and the beginning of the chapter on the analytic representation of functions and Fourier series. In addition, there were proofs from the second edition annotated by de La Vallée Poussin in preparation for the third.

From this material, Mawhin and Ponce undertook a genuine mathematical and editorial reconstruction of this "ghost edition." Wherever proofs from the third edition survive, they naturally serve as the principal source. Missing parts are completed using the annotated copy of the second edition and, when no new information is available, by reproducing the corresponding text of that edition. The editors therefore do not claim to reproduce exactly a volume that has been irretrievably lost, but rather to provide what may reasonably be regarded as a faithful representation of what the third edition would have been.

The interest of the resulting volume goes well beyond that of a bibliographical curiosity. It restores a particularly fertile moment in the history of analysis, when ideas that are now part of the standard education of an analyst were still in the process of being developed. More importantly, it allows the reader to observe a major mathematician absorbing, almost in real time, the deep transformations taking place in his field, confronting them with his own research, and reorganizing them into a form suitable for transmission through teaching. As the editors rightly point out, one rarely encounters an introductory analysis textbook containing so many original contributions by its own author. This is perhaps the most lasting value of the present reconstruction: it gives access not only to a lost chapter in the history of integration theory, but also to a particularly striking example of the intellectual assimilation and synthesis by which a great mathematician can transform a still-evolving field into a body of mathematics that is organized, intelligible, and teachable.

Reviewer: Frédéric Morneau-Guérin (Québec)

MSC:

01-06 Proceedings, conferences, collections, etc. pertaining to history and biography
01A55 History of mathematics in the 19th century
01A60 History of mathematics in the 20th century
26-03 History of real functions
00B15 Collections of articles of miscellaneous specific interest

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Biographic references:

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