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Inflation, Benchmark Return, Arbitrage and Asset Pricing

Claude Bergeron

School of Business Administration, Teluq University, Quebec (Quebec), Canada

École des sciences de l'administration, Université Téluq, 455, rue du Parvis, Québec (Québec), Canada, G1K 9H6

claude.bergeron@teluq.ca

Abstract

In this paper, we extend the benchmark asset pricing model of Bergeron (2022) and Baigent (2024), adding the factor of inflation. The extension model considers a standard discrete-time framework, and its central assumption supposes that the representative investor estimates, for each asset, the expected real return, as well as the expected relative return. To simplify the derivation, the extension model also considers the familiar *no-arbitrage* paradigm. Our main result indicates that the expected return of an asset is positively and linearly related to its *inflation beta* (given by the covariance between the asset's return and the inflation factor), in addition to its *benchmark beta* (obtained from the covariance between the asset's return and the benchmark factor). This suggests, from a theoretical point of view, that inflation and benchmark risks influence asset returns.

Keywords: Asset pricing, Inflation, Benchmark return, Arbitrage

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Abstract

In this paper, we extend the benchmark asset pricing model of Bergeron (2022) and Baigent (2024), adding the factor of inflation. The extension model considers a standard discrete-time framework, and its central assumption supposes that the representative investor estimates, for each asset, the expected real return, as well as the expected relative return. To simplify the derivation, the extension model also considers the familiar *no-arbitrage* paradigm. Our main result indicates that the expected return of an asset is positively and linearly related to its *inflation beta* (given by the covariance between the asset's return and the inflation factor), in addition to its *benchmark beta* (obtained from the covariance between the asset's return and the benchmark factor). This suggests, from a theoretical point of view, that inflation and benchmark risks influence asset returns.

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1. Introduction

Recently, Bergeron (2022) proposed a new asset pricing model based on the relative return to a benchmark, and Baigent (2024) provided a much simpler derivation of the Bergeron model, using no-arbitrage equilibrium conditions.

In this paper, we extend the benchmark model of Bergeron (2022) and Baigent (2024), adding the factor of inflation.

Many noteworthy studies have investigated the theoretical effects of inflation on asset returns. For example, following the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), Chen and Boness (1975) derived a simple formula characterizing the equilibrium relationship between risk and return, under the condition of uncertain inflation. Friend et al. (1976) derived a similar relationship, in a continuous-time framework. Elton et al. (1983) used arbitrage conditions to obtain an equilibrium real returns model. Along these lines, see Boudoukh and Richardson (1993), Kurisaki (2006), Katzur and Spierdijk (2013), and Boons et al. (2020), among others.

However, none of the above-mentioned studies uses a benchmark point of view to integrate the effects of inflation on asset pricing.

Our model development is based on the following assumptions. First, as in Bergeron (2022), we assume that the representative investor estimates, for each asset, the expected relative return to a benchmark. Second, as in Baigent (2024), we suppose that arbitrage opportunities are not possible, which implies the existence of the standard *stochastic discount factor* (see, for example, Cochrane, 2005, Chapter 4). Third, we establish that the representative investor also estimates, for each asset, the expected real return after inflation. Given this, our main result indicates that the expected return of an asset is positively and linearly related to its *inflation beta* (given by the covariance between the asset's return and the inflation factor), in addition to its *benchmark beta* (obtained from the covariance between the asset's return and the benchmark factor).

The primary motivation of this study is to integrate inflation into the benchmark model. Even so, another key motivation of this study is to identify the appropriate factors to consider in the multifactor risk-return relationship predicted by the arbitrage pricing theory (APT) of Ross (1976).

As noted by Campbell (2018, Chapter 3), the APT is appealing but has several weaknesses. For instance, the number of factors to use in the model is ambiguous. The APT does not itself reveal the identity of these factors, as well as the corresponding risk measures (*betas*). Moreover, the APT does not identify the signs or magnitudes of the risk prices (*lambdas*). Therefore, as mentioned by Campbell (2018), additional theoretical considerations are needed in order to select an appropriate K-factor model, and pin down the risk prices more precisely. Adopting our inflation–benchmark approach (with or without arbitrage conditions), our model reveals the following characteristics of the risk–return relationship: (1) within our proposed framework, the relevant predictor variables are the benchmark and inflation factors; (2) the risk measures correspond to the inflation and benchmark betas; (3) the sign of the risk prices is positive and their magnitudes increase with the volatility of the benchmark and inflation factors; and (4) the multidimensional risk-return relationship is linear.

In Section 2, we develop our inflation-benchmark model. In Section 3, we discuss the link between our model and the APT. In Section 4, we offer additional comments on our extension inflation-benchmark model, including the link between our extension model and the initial benchmark model. In Section 5, we conclude. A short demonstration is reported in the Appendix.

2. The inflation-benchmark model

In this section, we present our assumptions, and then derive the corresponding risk-return relationship.

The assumptions

By definition, we have:

$$1 + \textit{Real Return} \;=\; (1 + \textit{Absolute Return}) \,/\, (1 + \textit{Inflation Rate}),$$
$$1 + \textit{Relative Return} \;=\; (1 + \textit{Absolute Return}) \,/\, (1 + \textit{Benchmark Return}).$$

Our inflation-benchmark model is based on the previous definitions and the assumptions listed below:

A1 For each asset, the representative investor calculates the expected real return after inflation;

A2 For each asset, the representative investor also calculates the expected relative return to a benchmark;

A3 The representative investor seeks higher returns and avoids risk;

A4 Arbitrage opportunities are not possible.

Assumptions A1 and A2 are easily accepted if we acknowledge that a well-informed investor will attempt to estimate the real return on an investment and will also attempt to compare the investment return to a benchmark. Assumptions A3 and A4 are standard assumptions in finance (or economics). Here, we use these four unrestrictive assumptions to describe the risk-return relationship, under the condition of uncertain inflation. After adding the inflation factor, our model construction follows Bergeron (2022), and Baigent (2024).

The risk-return relationship

Given the available information at time t, we assume that a representative investor calculates the asset's expected real return (after inflation) and its expected relative return to a benchmark, in the following manner:

$$1 + \nu_{it} = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{\pi}_{t+1})\right], \tag{1}$$

$$1 + \mu_{it} = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{R}_{b,t+1})\right], \tag{2}$$

where $\tilde{R}_{i,t+1}$ is the (absolute) return of asset i ($i = 1, 2, \ldots, N$) at time $t + 1$, $\tilde{\pi}_{t+1}$ is the inflation rate at time $t + 1$, ν_{it} is the expected real return of asset i at time t, $\tilde{R}_{b,t+1}$ is the return of the benchmark portfolio b at time $t + 1$, and μ_{it} is the expected relative return of asset i at time t.[1] To simplify the notation, we can also express:

$$1 + \nu_{it} = E_t\left[\tilde{I}_{t+1}(1 + \tilde{R}_{i,t+1})\right], \tag{3}$$

[1] In this manuscript, the tilde (~) indicates a random variable. Operators E_t, V_t, and Cov_t refer respectively to mathematical expectation, variance and covariance, where index t implies that we consider the available information at time t.

$$1 + \mu_{it} = E_t\big[\tilde{B}_{t+1}(1 + \tilde{R}_{i,t+1})\big], \tag{4}$$

where $\tilde{I}_{t+1} \equiv 1/(1 + \tilde{\pi}_{1+t})$ represents the *Inflation factor* at time $t + 1$, and $\tilde{B}_{t+1} \equiv 1/(1 + \tilde{R}_{b,t+1})$ represents the *Benchmark factor* at time $t + 1$. Equation (3) plus (4) implies the following condition:

$$2 + \nu_{it} + \mu_{it} = E_t\big[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(1 + \tilde{R}_{i,t+1})\big]. \tag{5}$$

Dividing by two on each side of equation (5) gives:

$$1 + \nu_{it}/2 + \mu_{it}/2 = E_t\big[(1/2)(\tilde{I}_{t+1} + \tilde{B}_{t+1})(1 + \tilde{R}_{i,t+1})\big], \tag{6}$$

or, to simplify the notation:

$$1 + \theta_{it} = E_t\big[\tilde{F}_{t+1}(1 + \tilde{R}_{i,t+1})\big], \tag{7}$$

with $\theta_{it} \equiv (\nu_{it} + \mu_{it})/2$, and $\tilde{F}_{t+1} \equiv (\tilde{I}_{t+1} + \tilde{B}_{t+1})/2$. According to the *no-arbitrage paradigm* and the *law of one price*, the *fundamental equation of asset pricing* stipulates that (see Campbell, 2000, p. 1518):

$$1 = E_t\big[\tilde{S}_{t+1}(1 + \tilde{R}_{i,t+1})\big], \tag{8}$$

where $\tilde{S}_{t+1}$ represents the *Stochastic discount factor* (*SDF*) at time $t + 1$. So, if we (simply) accept the no-arbitrage paradigm, and assume that our *inflation-benchmark factor* $\tilde{F}_{t+1}$ corresponds to the usual *SDF* ($\tilde{F}_{t+1} = \tilde{S}_{t+1}$), then, in accordance with equations (7) and (8), we have:

$$1 = E_t\big[\tilde{F}_{t+1}(1 + \tilde{R}_{i,t+1})\big], \tag{9}$$

with θ_{it} equal to zero. Under the additional assumption that the inflation-benchmark factor coincides with the SDF, we also have, for the lowest-risk asset (identified by l):

$$1 = E_t\big[\tilde{F}_{t+1}(1 + \tilde{R}_{l,t+1})\big], \tag{10}$$

and equation (9) minus (10) yields:

$$0 = E_t\big[\tilde{F}_{t+1}(\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1})\big]. \tag{11}$$

From our definition of $\tilde{F}_{t+1}$, we can write:

$$0 = E_t\left[(1/2)(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1})\right], \tag{12}$$

which is equivalent to:

$$0 = E_t\left[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1})\right]. \tag{13}$$

From equation (13), the covariance definition implies that:

$$0 = Cov_t\left[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right] + E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]E_t\left[\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right]. \tag{14}$$

Isolating the asset return, we have:

$$E_t\left[\tilde{R}_{i,t+1}\right] = E_t\left[\tilde{R}_{l,t+1}\right] - \\ (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right], \tag{15}$$

and, rearranging, we get:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} - (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right], \tag{16}$$

with $\lambda_{ot} \equiv E_t\left[\tilde{R}_{l,t+1}\right] + Cov_t\left[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{l,t+1}\right]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. Employing basic covariance properties, and multiplying by the variance of the *Inflation factor*, $V_t[\tilde{I}_{t+1}]$, on both sides of equation (16), we can express:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{It}(-1)Cov_t\left[\tilde{I}_{t+1}, \tilde{R}_{i,t+1}\right]/V_t[\tilde{I}_{t+1}] - \\ (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right], \tag{17}$$

with $\lambda_{It} \equiv V_t[\tilde{I}_{t+1}]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. In the same manner, multiplying by the variance of the *Benchmark factor*, $V_t[\tilde{B}_{t+1}]$, on each side of equation (17), we can write:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{It}(-1)Cov_t\left[\tilde{I}_{t+1}, \tilde{R}_{i,t+1}\right]/V_t[\tilde{I}_{t+1}] + \\ \lambda_{Bt}(-1)Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right]/V_t[\tilde{B}_{t+1}], \tag{18}$$

with $\lambda_{Bt} \equiv V_t[\tilde{B}_{t+1}]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. Finally, using compact notation, we obtain the following linear relationship:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit}, \tag{19}$$

with $\beta_{Iit} \equiv -Cov_t\left[\tilde{I}_{t+1}, \tilde{R}_{i,t+1}\right]/V_t[\tilde{I}_{t+1}]$, and $\beta_{Bit} \equiv -Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right]/V_t[\tilde{B}_{t+1}]$.

Equation (19) represents our main result. This equation indicates that the asset's expected return is positively and linearly related to its *inflation beta* (β_{Iit}), in addition to its *benchmark beta* (β_{Bit}). Here, β_{Iit} is the *inflation beta* of asset i at time t. It measures the asset's return sensitivity to the inflation factor (given by the inflation rate). β_{Bit}, in turn, represents the *benchmark beta* of asset i at time t. It measures the asset's return sensitivity to the benchmark factor (given by the benchmark portfolio return). Since, by construction, we have $\tilde{I}_{t+1} \equiv 1/(1+\tilde{\pi}_{1+t})$ and a negative sign in the beta definition, it follows that a positive (negative) correlation between the asset's return and the inflation rate translates directly into a positive (negative) inflation beta. Likewise, a positive (negative) correlation between the asset's return and the benchmark return translates into a positive (negative) benchmark beta.

In equation (19), the asset's expected return is positively related to its betas because parameters λ_{It} and λ_{Bt} are mathematically greater than or equal to zero ($\lambda_{It} \geq 0$ and $\lambda_{Bt} \geq 0$). Indeed, by definition, we know that the variance of a variable is greater than or equal to zero. Also, by construction, we have $\tilde{I}_{t+1} \equiv 1/(1+\tilde{\pi}_{1+t})$, and $\tilde{B}_{t+1} \equiv 1/(1+\tilde{R}_{b,t+1})$, which implies that $\tilde{I}_{t+1} = C_t/\,\tilde{C}_{t+1}$ and $\tilde{B}_{t+1} = V_t/\,(\tilde{V}_{t+1} + \tilde{D}_{t+1})$, where C_t and $\tilde{C}_{t+1}$ represent the *consumer price index* at time t, and $t+1$, V_t and $\tilde{V}_{t+1}$ represent the *benchmark portfolio value* at time t and $t+1$, while $\tilde{D}_{t+1}$ is the benchmark dividend at $t+1$. As a result, parameters λ_{It} and λ_{Bt} are positive since all values included in their definitions are positive, that is: $V_t[\tilde{I}_{t+1}] \geq 0$; $V_t[\tilde{B}_{t+1}] \geq 0$; $\tilde{I}_{t+1} > 0$; and $\tilde{B}_{t+1} > 0$.

Also, in equation (19), the intercept parameter λ_{ot} corresponds to the expected return of the riskless asset plus a negative value, if the riskless asset's returns are negatively related to the inflation and benchmark factors. If the returns of the riskless asset are rather uncorrelated with these factors, then the intercept is straightly equivalent to its expected return value.

According to the usual *risk aversion postulate*, our assumption (A3) establishes that the representative investor seeks higher returns and avoids risk. This implies that the representative investor will require more return on a risky investment. Accepting our equation (19) and our interpretation of the inflation and benchmark betas (as discussed above), this implies that these two betas constitute the relevant measures of risk underlying the positive risk-return relationship of an asset. Moreover, within the proposed framework, the parameters $\lambda_{It}$and λ_{Bt} represent the appropriate prices of risk associated with the inflation and benchmark betas, respectively.

Overall, our main result demonstrates that the required rate of return on a risky asset is given by a linear combination of its inflation and benchmark risks. As we will see in the next section, this result appears to be fully consistent with the central prediction of the APT, when the return-generating process is a function of only two macroeconomic factors: benchmark return and inflation rate.

3. The inflation-benchmark model and the APT

In this section, we discuss the link between our inflation-benchmark model and the APT. This theory is based on arbitrage arguments and the following *law of one price*: two items which are the same cannot sell at different prices.

Given the available information at time t, the APT also assumes that the rate of return on any asset is a linear function of K factors, as shown below:

$$\tilde{R}_{i,t+1} = a_{it} + b_{1it}\tilde{F}_{1,t+1} + b_{2it}\tilde{F}_{2,t+1} + \cdots + b_{Kit}\tilde{F}_{K,t+1} + \tilde{e}_{i,t+1}, \quad (20)$$

with

$$0 = E_t\left[\tilde{e}_{i,t+1}\right] = Cov_t\left[\tilde{e}_{i,t+1}, \tilde{F}_{k,t+1}\right] = 0,$$

where a_{it} is the intercept for asset i at time t, $\tilde{F}_{k,t+1}$ is the factor k $(k = 1, 2, \dots, K)$ at time $t+1$, b_{kit} is the return sensitivity to factor k for asset i at time t, and $\tilde{e}_{i,t+1}$ is the usual random term for asset i at time $t+1$. In the APT, the returns generating process expressed in equation (20) represents an approximation of reality, and the factors to be integrated into the model are not determined by any economic theory (a priori). In equation (20), the K factors $\tilde{F}_{1,t+1}, \tilde{F}_{2,t+1}, \dots,$ and $\tilde{F}_{K,t+1}$ affect the returns on more than one asset. The parameter b_{1it}, b_{2it}, or b_{Kit}, is unique to each asset and represents an attribute of the asset that is considered a factor loading. The contribution of the APT is in demonstrating how one can go from this generating process to a description of a risk-return relationship under arbitrage conditions.

According to the APT, in equilibrium, all portfolios that can be selected from a set of assets that satisfy the conditions of (a) *using no wealth* and (b) *having no risk* must earn no return on average. More precisely, under arbitrage conditions, the APT shows that, algebraically, a set of $K+1$ coefficients $\lambda_{0t}, \lambda_{1t}, \lambda_{2t}, \dots, \lambda_{Kt}$ must exist such that:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{0t} + \lambda_{1t}b_{1it} + \lambda_{2t}b_{2it} + \cdots + \lambda_{Kt}b_{Kit}. \quad (21a)$$

If there is a riskless asset (F) with a riskless rate of return, then b_{kit} equals zero for all k, and equation (21a) implies that:

$$E_t[R_{F,t+1}] = \lambda_{0t},$$

where $R_{F,t+1}$ represents the (constant) return of the riskless asset at time $t+1$. Introducing the last value in equation (21a) also implies that:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} + \lambda_{1t}b_{1it} + \lambda_{2t}b_{2it} + \cdots + \lambda_{Kt}b_{Kit}. \quad (21b)$$

Here, parameters λ_{1t}, λ_{2t}, and λ_{Kt} express the price of risk for the corresponding factor sensitivities b_{1it}, b_{2it}, and b_{Kit}, respectively. If K portfolios exist such that each portfolio k has a unit sensitivity to the kth factor ($b_{kit} = 1$) and zero sensitivity to all other factors, then equation (21b) shows that:

$$E_t\left[\tilde{R}_{k,t+1}\right] = R_{F,t+1} + \lambda_{kt},$$

for all k ($k = 1,2, \dots, K$), where $\tilde{R}_{k,t+1}$ is the return of portfolio k at time $t+1$. Integrating the last result into equation (21b) allows us to express the main prediction of the APT in the following manner:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} + (\bar{R}_{1,t+1} - R_{F,t+1})b_{1it} + \cdots + (\bar{R}_{K,t+1} - R_{F,t+1})b_{Kit}, \quad (21c)$$

with $\bar{R}_{k,t+1} \equiv E_t\left[\tilde{R}_{k,t+1}\right]$. This reveals that each risk premium, λ_{kt}, corresponds to the difference between the expected value of a portfolio that has unit response to the kth factor, $\bar{R}_{k,t+1}$, and the riskless rate, $R_{F,t+1}$, that is to say: $\lambda_{kt} = \bar{R}_{k,t+1} - R_{F,t+1}$.

In light of this, theoretically, the APT requires the following implicit assumptions: (i) the return of an asset is correlated with K factors, and (ii) the return generating process defined by equation (20) is linear. In the previous section, our inflation-benchmark model demonstrates that the linearity assumption of the return generating process can be ignored. In addition, it demonstrates, from a theoretical point of view, how we can determine the appropriate *factor loadings* ($b_{1it}, b_{2it}, \dots,$ and b_{Kit}), as well as the appropriate *risk prices* ($\lambda_{1t}, \lambda_{2t}$ ..., and λ_{Kt}) that define a multifactor risk-return relationship, such as the APT and equations (21). In fact, from our inflation-benchmark model, formulated by equation (19), it is evident that: (1) the selected factors to use in a multifactor asset pricing are the inflation and benchmark factors (given by the inflation rate and benchmark return); and (2) the signs and magnitudes of the risk prices are given by parameters λ_{It} and λ_{Bt}, with positive signs, where the amplitude of the risk prices is higher when the variability of inflation and benchmark factors is greater. Additionally, our development shows that the linearity of the risk-return relationship predicted by our equation (19) is not simply a direct consequence of a predetermined assumption that presumes a linear return generating process. Within our framework, it is a mathematical result derived from the no-arbitrage postulate.

An enormous quantity of empirical studies have examined the main prediction of the APT. Many of these employed factor analysis or similar statistical methods. As noted by Campbell (2018, Chapter 3), this approach delivers little economic insight and tends to find more and more factors as the number of test assets increases. On top of that, a substantial body of literature has introduced market returns, inflation rates, and other macroeconomic variables into their empirical investigations of general multifactor risk models, with no theoretical restrictions. Examples include Chen et al. (1986), Fama and French (1993), Priestley (1996), Fama and French (2004, 2017), Barillas and Shanken (2018), Hou et al.

(2019), Li and Rao (2022), and Lönn and Schotman (2024). For instance, in their empirical study of the APT, Chen et al. (1986) show that several macroeconomic variables, including inflation, influence observed stock returns in the US. They find, among other things, that inflation is priced by the market, contrary to aggregate consumption. More generally, empirical evidence on the link between inflation, risk, and asset returns has also been documented (Priestley, 1996; Campbell and Vuolteenaho, 2004). Moreover, some studies, such as Wong and Wu (2003) and Chang (2013), find a positive correlation between stock returns and inflation (a positive beta), in contrast to Fama and Schwert (1977), Fama and Gibbons (1982), and Andrangi and Chatrath (2002), who report a negative relationship. Finally, Bansal and Shaliastovich (2013) argue that inflation affects asset returns because it is exposed to the same real shocks that drive consumption and long-run risk, while Duarte (2013) suggests that stocks whose returns covary negatively with inflation shocks earn higher unconditional returns.

More importantly for our purposes, the recent empirical study of Boons et al. (2020) shows that inflation risk constitutes a priced systematic factor in stock returns. In addition, the empirical results of Campbell et al. (2020) indicate that inflation shocks affect real bond returns and equity risk premia. Finally, the findings of Cieslak and Pflueger (2023) suggest that inflation has different implications for risk premia.

Nevertheless, the primary contribution of the present paper is strictly theoretical. It provides additional support for the inclusion of an inflation factor, as well as a benchmark factor, in the description of the risk–return relationship. Moreover, it reveals that the linearity result of the risk-return relationship, as predicted by our equation (19), cannot be attacked, in contrast to the standard APT, on a restrictive assumption that arbitrarily supposes (a priori) that the return generating process is (necessarily) linear, and given by unspecified factors.

4. Comments

In this section, we present some comments on our extension inflation-benchmark model. Our first comment discusses the link between our extension model and the initial benchmark model of Bergeron (2022) and Baigent (2024). Our second comment concerns the benchmark portfolio and risk-free rate. Our third comment is related to the classical CAPM and zero-beta CAPM. Our fourth comment is about the no-arbitrage postulate. Our fifth comment discusses two related inflation models. Our sixth comment concerns the lowest risk asset. Our seventh comment proposes practical applications. Our eighth comment explores future research. Our last comment addresses multifactor frameworks.

Comment 1: The initial benchmark model

It is very easy to demonstrate that the initial benchmark model could be viewed as a special case of our inflation-benchmark extension. In fact, assuming that the inflation rate is systematically equal to zero ($\tilde{\pi}_{1+t} = 0$), our main result, expressed by equation (19), can be reduced to the following expression:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{It}(0) + \lambda_{Bt}\beta_{Bit}, \tag{22a}$$

or, equivalently:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{Bt}\beta_{Bit}. \tag{22b}$$

Dividing by $V_t[\tilde{B}_{t+1}]$ on each side of equation (22b), we can write:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} - (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right], \tag{23}$$

and, in accordance with equation (16), we can also write:

$$E_t\left[\tilde{R}_{i,t+1}\right] = E_t\left[\tilde{R}_{l,t+1}\right] + (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{l,t+1}\right] - (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right]. \tag{24}$$

Subtracting $E_t\left[\tilde{R}_{l,t+1}\right]$ on each side of equation (24) allows us to express the following equality in terms of excess returns:

$$E_t\left[\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right] = (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{l,t+1}\right] - (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\right], \tag{25}$$

and knowing that $\tilde{\pi}_{1+t} = 0$, equation (25) becomes:

$$E_t\left[\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right] = Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{l,t+1} - \tilde{R}_{i,t+1}\right]/(1 + E_t[\tilde{B}_{t+1}]), \tag{26}$$

or, if we prefer:

$$E_t\left[\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right] = -Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right]/(1 + E_t[\tilde{B}_{t+1}]). \tag{27}$$

Similarly, for the benchmark portfolio (*b*), we have:

$$E_t\left[\tilde{R}_{b,t+1} - \tilde{R}_{l,t+1}\right] = -Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{b,t+1} - \tilde{R}_{l,t+1}\right]/(1 + E_t[\tilde{B}_{t+1}]). \tag{28}$$

Introducing equation (28) in (27), we find that:

$$E_t\left[\tilde{R}_{i,t+1}\right] = E_t\left[\tilde{R}_{l,t+1}\right] + E_t\left[\tilde{R}_{b,t+1} - \tilde{R}_{l,t+1}\right] \frac{Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{l,t+1}\right]}{Cov_t\left[\tilde{B}_{t+1}, \tilde{R}_{b,t+1} - \tilde{R}_{l,t+1}\right]}. \tag{29}$$

Equation (29) is exactly equivalent to the main prediction of the initial benchmark model as presented in equation (23) in Bergeron (2022). Equation (29) predicts that the asset's expected return is equal to the expected return of the lowest-risk security, plus a risk premium directly proportional to the covariance between the asset's excess return and the benchmark factor (identified by $\tilde{B}_{t+1}$).[2]

Comment 2: The benchmark portfolio and risk-free rate

In our setup, as in Bergeron (2022), a benchmark is simply a standard against which the performance of an asset can be evaluated. For instance, in the U.S. equity market, we can take the usual *S&P 500*, the *S&P 400 MidCap Index,* or the *S&P 600 SmallCap Index*. In this sense, there is an appropriate benchmark portfolio for each type of asset. Besides, in our framework, the lowest-risk asset does not have to be totally without risk. However, if we assume, for simplicity or practical application, that this asset is totally without risk and could be approximated by a Treasury bill, for example, then the value of parameter λ_{ot} will be equivalent to:

$$\lambda_{ot} = E_t\left[R_{F,t+1}\right] + Cov_t\left[\tilde{I}_{t+1} + \tilde{B}_{t+1}, R_{F,t+1}\right] / E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}], \tag{30}$$

or, equal to

$$\lambda_{ot} = E_t\left[R_{F,t+1}\right] + 0 = R_{F,t+1}, \tag{31}$$

where $R_{F,t+1}$ is (as previously noted) the return of the riskless asset. In this way, our main prediction, expressed by equation (19), will be such that:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} + \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit}, \tag{32a}$$

which is similar to the main prediction of the APT with the risk-free rate $R_{F,t+1}$, as formulated in section III by equation (21b).

If there exists a portfolio I, that has a unit sensitivity to the inflation factor ($\beta_{IIt} = 1$) and zero sensitivity to the benchmark, and if there exists a portfolio B, that has a unit sensitivity to the benchmark factor ($\beta_{BBt} = 1$) and zero sensitivity to inflation, then equation (32a) indicates that:

$$E_t\left[\tilde{R}_{I,t+1}\right] = R_{F,t+1} + \lambda_{It},$$

and that:

[2] See also equation (29) in Bergeron (2022), with the normality assumption, and equation (5) in Baigent (2024), under arbitrage conditions.

$$E_t\left[\tilde{R}_{B,t+1}\right] = R_{F,t+1} + \lambda_{Bt},$$

where $\tilde{R}_{I,t+1}$ is the return of portfolio I at time $t+1$, and $\tilde{R}_{B,t+1}$ is the return of portfolio B at time $t+1$. Integrating these results into equation (32a) allows us to express our main prediction in this manner:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} + (\bar{R}_{I,t+1} - R_{F,t+1})\beta_{Iit} + (\bar{R}_{B,t+1} - R_{F,t+1})\beta_{Bit}, \qquad (32b)$$

with $\bar{R}_{I,t+1} \equiv E_t\left[\tilde{R}_{I,t+1}\right]$, and $\bar{R}_{B,t+1} \equiv E_t\left[\tilde{R}_{B,t+1}\right]$, which is similar, this time, to the main prediction of the APT as formulated in section III by equation (21c).

Comment 3: The standard CAPM (or zero-beta CAPM) as a particular case

The standard CAPM can be viewed as a particular case of our main prediction if we accept the following restrictive assumptions: (1) a risk-free asset exists; (2) there is no inflation; (3) the benchmark portfolio corresponds to the market portfolio; and (4) asset and market returns are bivariate normally distributed. Indeed, from our equation (32a), we can write:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} + \lambda_{It}(0) + \lambda_{Bt}\beta_{Bit}, \qquad (33)$$

if we accept (again) that the inflation rate is systematically equal to zero ($\tilde{\pi}_{t+1} = 0$). Also, multiplying by $V_t[\tilde{B}_{t+1}]$ on each side of equation (33) indicates the following expression (see also equation (19)):

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} - Cov_t\left[\tilde{M}_{t+1}, \tilde{R}_{i,t+1}\right]/E_t\left[1 + \tilde{M}_{t+1}\right], \qquad (34)$$

if $\tilde{R}_{b,t+1} = \tilde{R}_{M,t+1}$, and $\tilde{B}_{t+1} = \tilde{M}_{t+1} \equiv 1/(1 + \tilde{R}_{M,t+1})$, where $\tilde{R}_{M,t+1}$ represents the market portfolio rate of return at time $t+1$. Now, according to Stein's lemma, we know that: if variable $\tilde{x}$ and variable $\tilde{y}$ are bivariate normal, the function $f(\tilde{x})$ is differentiable, and the expected of $f'(\tilde{x})$ is $< \infty$, then (see Cochrane, 2005, p. 164):

$$Cov[\tilde{y},\ f(\tilde{x})] = E[f'(\tilde{x})]Cov[\tilde{y},\ \tilde{x}].$$

So, if we suppose that $\tilde{R}_{M,t+1}$ and $\tilde{R}_{i,t+1}$ are bivariate normally distributed, then, from equation (34), we can write:

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F,t+1} - E_t\left[(-1)(1 + \tilde{R}_{M,t+1})^{-2}\right]Cov_t\left[1 + \tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}\right]/E_t\left[1 + \tilde{M}_{t+1}\right], \qquad (35)$$

or, equivalently:

$$E_t\big[\tilde{R}_{i,t+1}\big] = R_{F,t+1} + E_t\big[(1+\tilde{R}_{M,t+1})^{-2}\big]Cov_t\big[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}\big]/E_t\big[1+\widetilde{M}_{t+1}\big]. \quad (36)$$

In this manner, multiplying by $V_t[\tilde{R}_{M,t+1}]$ on each side of equation (36), we have:

$$E_t\big[\tilde{R}_{i,t+1}\big] = R_{F,t+1} + \lambda_{Mt}Cov_t\big[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}\big]/V_t[\tilde{R}_{Mt+1}], \quad (37)$$

with $\lambda_{Mt} \equiv V_t[\tilde{R}_{M,t+1}]E_t\big[(1+\tilde{R}_{M,t+1})^{-2}\big]/E_t\big[1+\widetilde{M}_{t+1}\big]$. Using compact notation, we can also write:

$$E_t\big[\tilde{R}_{i,t+1}\big] = R_{F,t+1} + \lambda_{Mt}\beta_{Mit}, \quad (38)$$

with $\beta_{Mit} \equiv Cov_t\big[\tilde{R}_{Mt+1}, \tilde{R}_{i,t+1}\big]/V_t[\tilde{R}_{M,t+1}]$, where β_{Mit} corresponds to the standard formulation of an asset's beta. For the market portfolio, equation (38) indicates that its expected rate of return is equal to:

$$E_t\big[\tilde{R}_{M,t+1}\big] = R_{F,t+1} + \lambda_{Mt}(1). \quad (39)$$

Therefore, introducing equation (39) in (38), we obtain:[3]

$$E_t\big[\tilde{R}_{i,t+1}\big] = R_{F,t+1} + (E_t\big[\tilde{R}_{M,t+1}\big] - R_{F,t+1})\beta_{Mit}. \quad (40)$$

Equation (40) reveals that the risk-return relationship predicted by the classical CAPM can be viewed as a special case of our main result, expressed by equation (19). In the Appendix, as proposed by Baigent (2024), we reproduce this development with the zero-beta portfolio, represented by the letter *z*, to show that:

$$E_t\big[\tilde{R}_{i,t+1}\big] = E_t\big[\tilde{R}_{z,t+1}\big] + (E_t\big[\tilde{R}_{M,t+1}\big] - E_t\big[\tilde{R}_{z,t+1}\big])\beta_{Mit}. \quad (41)$$

Equation (41) is here equivalent to the zero-beta CAPM of Black (1972).

Comment 4: The no-arbitrage paradigm

As noted by Fontana (2015), modern mathematical finance is strongly rooted in the no-arbitrage paradigm (see, for example, Platen, 2002; Kardaras, 2012; or Jarrow and Protter, 2011). The no-arbitrage paradigm excludes the possibility of *making money out of nothing* by cleverly trading in the financial market. In our model development, we referred to this principle in assuming that our *inflation-benchmark factor* is equivalent to the familiar

[3] Bergeron (2022) obtained a similar result using the familiar Taylor's theorem, but it allowed only an approximation.

stochastic discount factor, based on the *law of one price*, a pure *Arrow-Debreu security*, and the *no-arbitrage paradigm*. This proposed equality between the two factors represented an additional assumption, rather than a direct implication of no-arbitrage alone. However, as noted by Bergeron (2022), and others before, arbitrage opportunities are frequently visible in financial markets. Moreover, a pure *Arrow-Debreu security* has no strict equivalent in the real world, which makes it difficult to identify the *stochastic discount factor*. Nevertheless, we can derive our main prediction without any reference to arbitrage conditions if we simply accept the following assumption: the real and relative returns on any asset tend to be equal to the corresponding benchmark rates in the economy, plus a disturbance term equal to zero, on average.

For clarity, let $\tilde{R}_{i,t+1}^{Real}$ and $\tilde{R}_{i,t+1}^{Relative}$ be, respectively, the real and relative returns of asset i at time $t+1$, with $\tilde{R}_{i,t+1}^{Real} \equiv (1+\tilde{R}_{i,t+1})/(1+\tilde{\pi}_{t+1})-1$, and $\tilde{R}_{i,t+1}^{Relative} \equiv (1+\tilde{R}_{i,t+1})/(1+\tilde{R}_{b,t+1})-1$. For the benchmark portfolio, represented by the letter *b*, we thus have: $\tilde{R}_{b,t+1}^{Real} \equiv (1+\tilde{R}_{b,t+1})/(1+\tilde{\pi}_{t+1})-1$, and $\tilde{R}_{b,t+1}^{Relative} \equiv (1+\tilde{R}_{b,t+1})/(1+\tilde{R}_{b,t+1})-1 = 0$. Here, given the available information at time t, we assume the validity of the following *inflation-benchmark process*:

$$\tilde{R}_{i,t+1}^{Real} = \tilde{R}_{b,t+1}^{Real} + \tilde{\varepsilon}_{i,t+1}^{Real}, \tag{42a}$$

$$\tilde{R}_{i,t+1}^{Relative} = \tilde{\varepsilon}_{i,t+1}^{Relative}, \tag{42b}$$

$$E_t[\tilde{\varepsilon}_{i,t+1}^{Real}] = E_t[\tilde{\varepsilon}_{i,t+1}^{Relative}] = 0, \tag{42c}$$

where $\tilde{\varepsilon}_{i,t+1}^{Real}$ and $\tilde{\varepsilon}_{i,t+1}^{Relative}$ are the usual random error terms, associated with $\tilde{R}_{i,t+1}^{Real}$ and $\tilde{R}_{i,t+1}^{Relative}$, respectively. Taking the expected value of each side of equations (42a) and (42b), we get:

$$E_t\left[\tilde{R}_{i,t+1}^{Real}\right] = E_t\left[\tilde{R}_{b,t+1}^{Real}\right], \tag{43a}$$

$$E_t\left[\tilde{R}_{i,t+1}^{Relative}\right] = 0. \tag{43b}$$

Our *inflation-benchmark process*, described above, simply suggests that, for different states of nature and probabilities, the real and relative returns of an asset (such as a stock) will sometimes be superior and sometimes inferior to the corresponding benchmark rates in the economy (viewed as reference values). For example, if the benchmark value for the real return on the stock market is 6%, then, in accordance with our process above, the equivalent rate on any stock should fluctuate at around 6%. In the same manner, knowing that the relative return for the benchmark portfolio is, by definition, equal to zero, then, in accordance with our process above, the equivalent rate on any stock should fluctuate at around 0%. As our previous development suggests, and as we will show below, this point of view is fully consistent with the risk-aversion postulate and a positive relationship between risk and return, described in nominal terms. In fact, using the notation above in our initial equations (1) and (2), we can write:

$$1 + E_t\left[\tilde{R}_{i,t+1}^{Real}\right] = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{\pi}_{t+1})\right], \tag{44}$$

$$1 + E_t\left[\tilde{R}_{i,t+1}^{Relative}\right] = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{R}_{b,t+1})\right], \tag{45}$$

and accepting our inflation-benchmark process, equations (44) and (45) become:

$$1 + E_t\left[\tilde{R}_{b,t+1}^{Real}\right] = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{\pi}_{t+1})\right], \tag{46}$$

$$1 = E_t\left[(1 + \tilde{R}_{i,t+1})/(1 + \tilde{R}_{b,t+1})\right], \tag{47}$$

or, equivalently:

$$1 + E_t\left[\tilde{R}_{b,t+1}^{Real}\right] = E_t\left[\tilde{I}_{t+1}(1 + \tilde{R}_{i,t+1})\right], \tag{48}$$

$$1 = E_t\left[\tilde{B}_{t+1}(1 + \tilde{R}_{i,t+1})\right]. \tag{49}$$

Thus, equation (48) plus (49) gives

$$2 + E_t\left[\tilde{R}_{b,t+1}^{Real}\right] = E_t\left[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(1 + \tilde{R}_{i,t+1})\right], \tag{50}$$

and for the lowest-risk asset, included in the benchmark portfolio, we get:

$$2 + E_t\left[\tilde{R}_{b,t+1}^{Real}\right] = E_t\left[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(1 + \tilde{R}_{l,t+1})\right], \tag{51}$$

while equation (50) minus (51), indicates that:

$$E_t\left[\tilde{R}_{b,t+1}^{Real}\right] - E_t\left[\tilde{R}_{b,t+1}^{Real}\right] = 0 = E_t\left[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{l,t+1})\right], \tag{52}$$

which is equivalent to our previous equation (13). Therefore, if we replicate our above development between equations (14) and (18), we will again find our main result, expressed by equation (19), with no reference to the *stochastic discount factor*, or *arbitrage conditions*.

In mathematical finance, a key field of research focuses on no-arbitrage conditions weaker than the classical restrictive notion of no-arbitrage that postulates, for example, the following proposition: *No free lunch with vanishing risk*. More particularly, we have the influential *benchmark approach* of Platen (2006) and Platen and Heath (2006). This approach is based on the *growth optimal portfolio* of Kelly (1956) and the *numeraire portfolio* of Long (1990). It implies accepting the natural proposition that investors prefer more rather than less, in a continuous-time framework. In this set-up, the CAPM follows without expected utility functions, *Markovianity*, equilibrium assumptions, or arbitrage conditions.

In the same vein, several studies confirm the popularity of the benchmark approach. For instance, we have Karatzas and Kardaras (2007), Platen and Rendek (2012), Du and Platen (2016), Curchiero et al. (2019), and Bauer (2023). Like benchmark asset pricing models, our above procedure characterizes the risk-return relationship without utility functions, equilibrium assumptions, or arbitrage conditions. Nonetheless, contrary to Platen (2006) and others, our asset pricing framework does not require, as in Bergeron (2022), these restrictive assumptions: (1) security price movements are continuous and random; (2) random fluctuations are described by independent Wiener processes. This means that our approach (in this section) makes no assumptions regarding the diffusion process, as well as arbitrage conditions. It considers a standard discrete-time framework, and its central postulates simply recognize the estimation of the expected real and relative returns, in addition to our inflation-benchmark process, assumed by equations (42). In this regard, our model offers comparable results to the continuous benchmark approach, but in an easier mathematical framework, adding the factor of inflation. Moreover, it allows us to avoid two restrictive assumptions required by the benchmark approach.

Besides, the fact that our last procedure ignores any arbitrage restriction does not mean that we reject the arbitrage paradigm. Rather, it simply means that we do not (absolutely) need to introduce any particular arbitrage restriction to obtain a useful risk-return relationship (under the condition of inflation).

Comment 5: Two related inflation models

The *inflation-arbitrage model* of Elton et al. (1983) has some similarities with ours. Their model, based on the APT, shows that if the process generating real returns is affected by both the market return and inflation, then a model for equilibrium real returns can be derived. Like ours, their paper is strictly theoretical (without any empirical observations). In addition, their framework begins by using the real return on a security (see their equation 1, at p. 527), and expresses their model's main predictions in nominal terms or absolute returns (see their equation 14 or 15 in section III). However, our model also differs significantly from that proposed by Elton et al. (1983). First, in our framework, we do not make the following restrictive assumption: the return generating process is linear and limited to the return on the market and inflation rate.[4] Second, in our procedure, we do not (necessarily) assume the existence of a risk-free asset, nor a zero-covariance portfolio. More importantly, our main prediction reveals different beta risk measures, compared to the initial *inflation-arbitrage model*.

In the same way, the model of Bergeron (2014) also examines the theoretical effects of inflation and risk on asset returns. However, the key differences are as follows. First, the model of Bergeron (2014) is based on the main prediction of the standard CAPM (see equation 1, in section 2), and there is no link with the no-arbitrage condition. Second, this model assumes the existence of a zero-beta portfolio (as others). Third, our present framework adopts a benchmark procedure, using relative returns, which leads to a distinct risk-return relationship. Moreover, it leads to different predictions if we use the appropriate benchmark index for each asset class.

Comment 6: The model without lowest-risk asset

In practice, the lowest-risk asset is unknown, a priori. This situation is similar to the zero-beta portfolio, which is unobservable on financial markets and leads to considerable empirical difficulties, as recently pointed out by Beaulieu et al. (2013), Beaulieu et al. (2022) and Beaulieu et al. (2023). On the other hand, a *pure* risk-free security can be viewed as unrealistic, since all financial investment carry some degree of uncertainty. In this section, we rederive our inflation-benchmark relationship without any reference to a lowest-risk asset, zero-beta portfolio or pure riskless security. Our derivation excludes any use of the following values: $\tilde{R}_{l,t+1}$, $\tilde{R}_{z,t+1}$, or $R_{F,t+1}$. Indeed, our equation (9), which is similar in form to the *fundamental equation of asset pricing*, indicates, for the benchmark portfolio (b), that:

$$1 = E_t\big[\tilde{F}_{t+1}(1 + \tilde{R}_{b,t+1})\big]. \tag{53}$$

Therefore, equation (9) minus (53), exhibits the following new condition:

$$0 = E_t\big[\tilde{F}_{t+1}(\tilde{R}_{i,t+1} - \tilde{R}_{b,t+1})\big]. \tag{54}$$

From our definition of $\tilde{F}_{t+1}$, we can express:

$$0 = E_t\big[(1/2)(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{b,t+1})\big], \tag{55}$$

or, alternately:

[4] Note that any return-generating process also makes different restrictive assumptions related to the usual random term (e).

$$0 = E_t\big[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{b,t+1})\big]. \tag{56}$$

From equation (56), the covariance definition now implies that:

$$0 = Cov_t\big[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{b,t+1}\big] + E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]E_t\big[\tilde{R}_{i,t+1} - \tilde{R}_{b,t+1}\big]. \tag{57}$$

Isolating the return of the asset, we have, this time:

$$E_t\big[\tilde{R}_{i,t+1}\big] = E_t\big[\tilde{R}_{b,t+1}\big] - \\ (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\big[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{b,t+1}\big], \tag{58}$$

or, after simple manipulations:

$$E_t\big[\tilde{R}_{i,t+1}\big] = \lambda^*_{ot} - (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\big[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{i,t+1}\big], \tag{59}$$

with $\lambda^*_{ot} \equiv E_t\big[\tilde{R}_{b,t+1}\big] + Cov_t\big[\tilde{I}_{t+1} + \tilde{B}_{t+1}, \tilde{R}_{b,t+1}\big]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. Employing basic covariance properties, and multiplying by the variance of the *Inflation factor*, on both sides of equation (59), we can now express:

$$E_t\big[\tilde{R}_{i,t+1}\big] = \lambda^*_{ot} + \lambda_{It}(-1)Cov_t\big[\tilde{I}_{t+1}, \tilde{R}_{i,t+1}\big]/V_t[\tilde{I}_{t+1}] - \\ (1/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}])Cov_t\big[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\big], \tag{60}$$

where λ_{It} is still equivalent to $V_t[\tilde{I}_{t+1}]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. In the same way, multiplying by the variance of the *Benchmark factor*, on each side of equation (60), we can also write:

$$E_t\big[\tilde{R}_{i,t+1}\big] = \lambda^*_{ot} + \lambda_{It}(-1)Cov_t\big[\tilde{I}_{t+1}, \tilde{R}_{i,t+1}\big]/V_t[\tilde{I}_{t+1}] + \\ \lambda_{Bt}(-1)Cov_t\big[\tilde{B}_{t+1}, \tilde{R}_{i,t+1}\big]/V_t[\tilde{B}_{t+1}], \tag{61}$$

where λ_{Bt} is still equivalent to $V_t[\tilde{B}_{t+1}]/E_t[\tilde{I}_{t+1} + \tilde{B}_{t+1}]$. Finally, using compact notation, we obtain the following new linear relationship:

$$E_t\big[\tilde{R}_{i,t+1}\big] = \lambda^*_{ot} + \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit}, \tag{62}$$

where β_{Iit} and β_{Bit} are the same as before. As our main result, equation (62) indicates that the expected return of an asset is positively and linearly related to its *inflation beta*, in addition to its *benchmark beta*. The only difference between equation (62) and our previous equation (19) is the intercept λ^*_{ot}, distinct from our previous intercept λ_{ot}. Intercept λ_{ot} was defined by the lowest-risk asset (and $\tilde{R}_{l,t+1}$), while λ^*_{ot} is defined by the benchmark portfolio (and $\tilde{R}_{b,t+1}$).

Our last result formulated by equation (62) offers several advantages over any asset pricing model that integrates either $\tilde{R}_{l,t+1}$, $\tilde{R}_{z,t+1}$ or $R_{F,t+1}$. In theory, this result is freed of the standard restrictive assumptions related to the risk-free asset and zero-beta portfolio. It also avoids the implicit assumption that the lowest-risk asset is identifiable (or known). In practice, it simplifies estimation of an asset's required return, in a context of inflation, since in our last equation the only macroeconomic variables to be selected and calculated are the benchmark index return (for $\tilde{R}_{b,t+1}$), and the inflation rate (for $\tilde{I}_{t+1}$), with no selection required for R_{F+1}, $\tilde{R}_{z,t+1}$, or $\tilde{R}_{l,t+1}$. The riskless rate is presumed to be fixed, but in reality, it fluctuates. Its estimation thus implies an additional source of errors for real investors. Moreover, these potential errors are greater for the zero-beta portfolio, as well as the lowest-risk asset, since such funds are not traded on financial markets.

This last result can also be obtained using our inflation-benchmark process, assumed by equations (42a), (42b), and (43c), without any conditions related to the no-arbitrage principle. Indeed, for the benchmark portfolio (*b*), equation (50) indicates (in *Comment 4*) the equality below:

$$2 + E_t\big[\tilde{R}_{b,t+1}^{Real}\big] = E_t\big[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(1 + \tilde{R}_{b,t+1})\big], \tag{63}$$

while equation (50) minus (63) gives:

$$E_t\big[\tilde{R}_{b,t+1}^{Real}\big] - E_t\big[\tilde{R}_{b,t+1}^{Real}\big] = 0 = E_t\big[(\tilde{I}_{t+1} + \tilde{B}_{t+1})(\tilde{R}_{i,t+1} - \tilde{R}_{b,t+1})\big], \tag{64}$$

which is equivalent to our equation (56), above. If we replicate our development between equations (57) and (62), we will again find our previous result, formulated by equations (62), with no reference to the *stochastic discount factor*, or *arbitrage conditions*.

It is also easy to demonstrate that our initial result represents a particular case of our last result, if we integrate the additional restrictive assumption related to a risk-free asset. In fact, for the riskless asset (*F*), equation (62) directly indicates the following reduced expression:

$$E_t\big[R_{F,t+1}\big] = \lambda_{ot}^{*} + \lambda_{It}(0) + \lambda_{Bt}(0). \tag{65}$$

Thus, equation (62) minus (65) gives:

$$E_t\big[\tilde{R}_{i,t+1}\big] - R_{F,t+1} = \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit}, \tag{66}$$

which is identical to our initial result when the lowest-risk asset (*l*) is absolutely certain, as expressed by equation (32a), in *Comment 2*.

Comment 7: Discussion of the practical implications for finance

Our model offers at least two practical implications for finance. First, we can use our model to evaluate portfolio performances. Second, we can apply our main result to estimate the cost of equity capital.

Indeed, as noted by Fama and French (2004), and Bergeron (2025), finance textbooks often recommend using the classical CAPM to measure the performance of mutual funds and other managed portfolios. However, as pointed out by Fama and French (2004, p. 37) market beta is not a complete description of an asset's risk and *the search should turn to asset pricing models that can do a better job* (see also Campbell, 2018, Chapter 3). Here, we propose utilizing our main prediction to appraise the realized performance of a stock portfolio by integrating an inflation risk measure, in addition to the common beta. The procedure is based on the following development. From our main result, expressed by equation (19), we can write:

$$0 = E_t\big[\tilde{R}_{i,t+1}\big] - [\lambda_{ot} + \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit}], \tag{67}$$

and in accordance with equation (67), we have:

$$0 = E_t\big[\tilde{R}_{P,t+1}\big] - [\lambda_{ot} + \lambda_{It}\beta_{IPt} + \lambda_{Bt}\beta_{BPt}], \tag{68}$$

where the index *P* denotes a stock portfolio. Therefore, adapting the well-known *Jensen's Alpha* to equation (68), we can reasonably suggest that:

$$\alpha_{Pt} = R_{Pt} - [\lambda_{ot} + \lambda_{It}\beta_{IPt} + \lambda_{Bt}\beta_{BPt}], \tag{69}$$

where α_{Pt} is defined as the *estimated alpha* of portfolio P, at time t, while R_{Pt} is the realized return of portfolio P, between time $t-1$ and t. Given this, the estimated alpha value will represent a risk-adjusted performance metric that calculates the excess return of a stock portfolio over its normal return, as predicted by our inflation-benchmark model. A positive alpha will indicate outperformance, while a negative alpha will indicate underperformance. In short, the procedure described above offers an additional useful tool for assessing the performance of a stock portfolio that accounts for both inflation and market risks.

Furthermore, as noted by Fama and French (2004), the CAPM is also often used to estimate the cost of equity capital. This time, the prescription is to determine the risk-free rate, calculate the stock's market beta, and integrate these values into the CAPM to approximate the firm's cost of equity. Here, we propose using our main prediction to obtain a similar estimation with two risk measures. Our proposed procedure is the following. From equation (19), ignoring the index of time, we can express:

$$E[\tilde{R}_i] = \lambda_o + \lambda_I \beta_{Ii} + \lambda_B \beta_{Bi}, \tag{70}$$

and establish that:

$$k_i = \lambda_o + \lambda_I \beta_{Ii} + \lambda_B \beta_{Bi}, \tag{71}$$

where the index *i* denotes the firm i, and where k_i represents the approximate cost of equity (of firm i, associated with stock i). Adopting a typical approach, parameter λ_o can be determined by the rate of return on a Treasury bill (for example), while parameters λ_I and λ_B can be estimated with familiar cross-sectional regressions. For the approximate *benchmark beta*, β_{Bi}, we can conduct a simple regression between the stock and the benchmark returns, using a common index such as the S&P 500, the S&P 400 MidCap Index, or the S&P 600 SmallCap Index. Finally, for the approximate *inflation beta*, β_{Bi}, we can regress the realized returns of the stock on the corresponding observed inflation rates. In that manner, in practice, the resulting cost of equity will incorporate both inflation and market risks as well.

Comment 8: Some suggestions for future research

In this brief comment, we make some suggestions for future research in the field of inflation-related asset pricing. These suggestions are connected to (1) the *relative-beta* model, (2) the aggregate consumption, and (3) a potential empirical test.

The first proposal is to combine our inflation-benchmark model with the relative-beta approach initiated by Bergeron (2025). As mentioned by Bergeron (2025), the classical CAPM and its most influential extensions[5] all predict that the expected return of a risky asset (ER) is equal to the risk-free rate (R_F), or the expected return of a zero-beta portfolio (ER_Z), plus a positive risk premium. This is illustrated as follows:

$$ER = R_F \; (or \; ER_Z) + risk \; premium.$$

The relative-beta model, for its part, puts forward a complementary method. Instead of beginning the estimation with the lowest return ($R_F \; or \; ER_Z$), the method begins the estimation with the average market return and then adds a positive or negative risk adjustment. To be more precise, the method uses the following adjustment:

$$ER = ER_M \pm \; risk \; adjustment,$$

[5] See, for example, Black (1972), Merton (1972), Ross (1976), Breeden (1979), Epstein and Zin (1989), Fama and French (1993), Campbell and Vuolteenaho (2004), and Campbell et al. (2018).

where ER_M represents the expected return of the market portfolio, regarded as a benchmark. This procedure is motivated by the fact that the future is uncertain, and that all financial investments carry some degree of risk. It is also motivated by the difficulty of identifying a good proxy for the risk-free rate or the zero-beta return. Given that, our suggestion for a future theoretical study is to show how we can estimate the *risk adjustment* above with our inflation-benchmark model. More particularly, the inflation-benchmark extension should be such that:

$$risk\ adjustment = inflation\ adjustment + benchmark\ adjustment,$$

where the inflation and benchmark adjustments are obtained from the inflation and benchmark betas, derived with our framework.

Our second future research proposal is to integrate the *consumption risk* into our inflation-benchmark model. This proposal relies on the classical Consumption-CAPM of Breeden (1979). Briefly, our suggestion for another future theoretical study is to demonstrate how we can derive a three-factor extension with consumption, in addition to inflation and benchmark. This time, the extension model should be based on the following useful value:

1 + *Growth-Adjusted Return* = (1 + *Absolute Return*) / (1 + *Consumption Growth*),

where *Consumption Growth* corresponds to the aggregate consumption growth rate.[6] Employing the above definition, the derivation of the extension model should include the following additional assumption (along with A1 to A4):

A5 For each asset, the representative investor also calculates the expected growth-adjusted return.

In this manner, the derivation of a three-factor model will be like the present derivation, except that the additional constraint, given by the growth-adjusted return, introduces another factor. At the end, the new prediction should be similar to this new relation:

$$E_t\left[\tilde{R}_{i,t+1}\right] = \lambda_{ot} + \lambda_{It}\beta_{Iit} + \lambda_{Bt}\beta_{Bit} + \lambda_{Ct}\beta_{Cit}, \tag{72}$$

where β_{Cit} is defined as the *consumption beta* of asset i at time t, while λ_{Ct} is the consumption price of risk, at time t. Overall, this suggestion for future research represents a potential novel extension of the Canonical Consumption-CAPM, as well as an extension of our inflation-benchmark model.

[6] See Lettau and Ludvigson (2001) or Cochrane (2011).

Our third future research proposal is to conduct an empirical investigation of our three-factor model, as expressed by equation (72). Ignoring the index t, the empirical test could be grounded in the following prediction:

$$E[\tilde{R}_i] = \lambda_o + \lambda_I \beta_{Ii} + \lambda_B \beta_{Bi} + \lambda_C \beta_{Ci}. \tag{73}$$

The approach, for example, could consist of regressing a cross-section of average asset returns on the historical betas, as shown below:

$$\bar{R}_i = \lambda_o + \lambda_I \hat{\beta}_{Ii} + \lambda_B \hat{\beta}_{Bi} + \lambda_C \hat{\beta}_{Ci}, \tag{74}$$

where $\bar{R}_i$ is the historical average return of stock i, while $\hat{\beta}_{Ii}$, $\hat{\beta}_{Bi}$ and $\hat{\beta}_{Ci}$ are the corresponding empirical estimators of β_{Ii}, β_{Bi} and β_{Ci}, for stock i. In short, the investigation will consist of comparing the resulting cross-sectional coefficients with the theoretical lambdas (λ_o, λ_I, λ_B and λ_C).

Comment 9: Existing multifactor frameworks

This comment clarifies how our inflation–benchmark model differs from existing multifactor frameworks.

Following the classical CAPM, the intertemporal approach of Merton (1973) is often regarded as the first major multifactor asset pricing model. In this framework, the market portfolio and a set of (unidentified) state variables act as the relevant pricing factors. These factors arise from investors' demand to hedge against future risky investment opportunities. The model shows that the expected excess return on any asset depends on its exposure to the market portfolio, as well as on several state-variable betas. In the APT of Ross (1976), asset returns are driven by multiple (also unidentified) macroeconomic factors. From this return-generating process, Ross demonstrates that the absence of arbitrage implies that expected asset returns are linear functions of the corresponding factor sensitivities. In the three-factor model of Fama and French (1993), asset returns are driven by three explicit factors: the market, size, and book-to-market portfolio returns. This approach builds on earlier empirical evidence documenting systematic relations between stock returns, firm size, and book-to-market equity. It yields a linear risk–return relationship characterized by three beta measures.

Besides, as noted by Campbell (2000, p. 1525), if one assumes that the stochastic discount factor is a linear combination of K common factors, then a multifactor pricing model follows. When empirical factors are used, this approach is adopted by Cochrane (1996), Jagannathan and Wang (1996), and Lettau and Ludvigson (2001). The studies of Pàstor and Stambaugh (2002), Lawrence et al. (2007), Hou et al. (2011), Fama and French

(2017), and Cooper et al. (2021) further confirm the relevance of a multidimensional approach.

However, our inflation–benchmark model differs significantly from the multifactor frameworks discussed above. First, none of these significant contributions is grounded in the benchmark framework developed by Bergeron (2022) and Baigent (2024). Second, none of these models relies on the core assumption that the representative investor evaluates expected real and relative returns. Third, none of these frameworks uses the return on the lowest-risk asset (R_l) to relax the restrictive assumption of the existence of either a risk-free asset or a zero-beta portfolio. Finally, none of these approaches derives both the number and the appropriate measures of systematic risk strictly from theoretical arguments.

5. Conclusions

As noted by Cieslak and Pflueger (2023), the recent surge in inflation has spurred renewed questions about this macroeconomic force (see also Bergeron, 2024). In this paper, we developed, in a standard discrete-time framework, a simple asset pricing model that integrates inflation and benchmark factors. We began by assuming that a risk-averse investor estimates the expected real and relative returns for each asset. Thereafter, we demonstrated that the expected return of an asset is positively and linearly related to its *inflation beta*, in addition to its *benchmark beta*. Thereby, we suggested that the *inflation* and *benchmark betas* represent the appropriate measures of risk that characterize the theoretical relationship between risk and return. Moreover, we showed that this result can be obtained with or without arbitrage restrictions.

As such, the main contributions of this paper may be summarized as follows. First, it demonstrates that we can add inflation to the benchmark model of Bergeron (2022) and Baigent (2024). Second, it provides additional support for the introduction of inflation in asset pricing. Third, it reveals that the linearity assumption of the return generating process can be relaxed in a multifactor model such as the APT. Fourth, it precisely describes which *factor loadings* and *risk prices* define the multidimensional relationship between risk and expected return.

Many empirical studies have already examined the effects of inflation on financial markets. Similarly, numerous analyses have investigated the cross-section of historical asset returns, with or without accounting for inflation.[7] The present paper excludes any tests and empirical analysis (as many important articles in finance or economics). Its global contribution is essentially theoretical. Using a solid framework, this paper demonstrates that inflation and benchmark betas influence asset returns.

[7] See, again, Campbell (2018, Chapter 3).

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Appendix

In this appendix, we demonstrate that the zero-beta CAPM of Black (1972) can be viewed as a special case of our inflation-benchmark model if we accept the following restrictive assumptions: (1) a zero-beta portfolio exists; (2) there is no inflation; (3) the benchmark portfolio corresponds to the market portfolio; and (4) asset and market returns are bivariate normally distributed. Indeed, if $\tilde{R}_{l,t+1} = \tilde{R}_{z,t+1}$; $\tilde{R}_{b,t+1} = \tilde{R}_{M,t+1}$; and $\tilde{\pi}_{1+t} = 0$, then $\tilde{I}_{t+1} = 1$, and $\tilde{B}_{t+1} = \tilde{M}_{t+1} \equiv 1/(1 + \tilde{R}_{M,t+1})$. Therefore, in accordance with our initial definition of parameter λ_{ot}, we get:

$$\lambda_{ot} = E_t[\tilde{R}_{z,t+1}] + Cov_t[1 + \tilde{M}_{t+1}, \tilde{R}_{z,t+1}]/E_t[1 + \tilde{M}_{t+1}] = E_t[\tilde{R}_{z,t+1}], \quad \text{(A1)}$$

and our main result, expressed by equation (19), is equivalent to:

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{z,t+1}] + \lambda_{Bt}\beta_{Bit}. \quad \text{(A2)}$$

Multiplying by $V_t[\tilde{B}_{t+1}]$ on each side of equation (A2) gives:

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{z,t+1}] - Cov_t[\tilde{M}_{t+1}, \tilde{R}_{i,t+1}]/E_t[1 + \tilde{M}_{t+1}], \quad \text{(A3)}$$

and assuming that $\tilde{R}_{M,t+1}$ and $\tilde{R}_{i,t+1}$ are bivariate normally distributed, we have:

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{z,t+1}] - E_t[(-1)(1 + \tilde{R}_{M,t+1})^{-2}]\, Cov_t[1 + \tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}]/E_t[1 + \tilde{M}_{t+1}], \quad \text{(A4)}$$

or, equivalently:

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{z,t+1}] + E_t[(1 + \tilde{R}_{M,t+1})^{-2}]\, Cov_t[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}]/E_t[1 + \tilde{M}_{t+1}]. \quad \text{(A5)}$$

For the market portfolio (*M*), equation (A5) indicates:

$$E_t[\tilde{R}_{M,t+1}] = E_t[\tilde{R}_{z,t+1}] + E_t[(1 + \tilde{R}_{M,t+1})^{-2}]V_t[\tilde{R}_{M,t+1}]/E_t[1 + \tilde{M}_{t+1}], \quad \text{(A6)}$$

and introducing equation (A6) in (A5), we finally get, after simple manipulations:

$$E_t\big[\tilde{R}_{i,t+1}\big] = E_t\big[\tilde{R}_{z,t+1}\big] + (E_t\big[\tilde{R}_{M,t+1}\big] - E_t\big[\tilde{R}_{z,t+1}\big])\frac{Cov_t[\tilde{R}_{M,t+1},\tilde{R}_{i,t+1}]}{V_t[\tilde{R}_{M,t+1}]}, \qquad \text{(A7)}$$

the zero-beta CAPM.