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Chebyshev centers and radii for sets induced by quadratic matrix inequalities. (English. English summary)

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Given a compact family of matrices equipped in a space equipped with norm, the center and radius of the smallest enclosing ball are known as the Chebyshev center and Chebyshev radius, respectively. The maximal distance between any two elements of the set defines its diameter.

The computational difficulty of determining the Chebyshev center and Chebyshev radius is strongly influenced by both the geometry of the set and the choice of norm.

In the present work, the authors derive explicit closed-form expressions for the Chebyshev center, Chebyshev radius, and diameter for a specific class of compact sets generated by quadratic matrix inequalities (QMIs). They further investigate the radius of the largest ball contained in such QMI-defined sets. When the interior of the set is empty, they instead characterize the radius of the largest ball of lower dimension that can be embedded in the set. All results are obtained for arbitrary unitarily invariant matrix norms, formulated via symmetric gauge functions.

The paper is organized as follows.

Section 2 is devoted to (notational and conceptual) preliminaries.

Section 3 includes the main result: a theorem providing closed-form expressions for the Chebyshev center, Chebyshev radius, and the diameter of certain types of QMI-induced sets with respect to an arbitrary unitarily invariant matrix norm.

Section 4 discusses in detail the applications of the presented results in data-driven modeling and control of unknown linear time-invariant systems using a set-membership approach. Several explicit examples are given, with supporting calculations and figures.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.