In special cases, **eigenvalues** of $n \times n$ **bistochastic** matrices lie in a **regular n-gon** in $\mathbb{D}$.

Which complex numbers can be eigenvalues of $n \times n$ bistochastic matrices?

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1 Intro

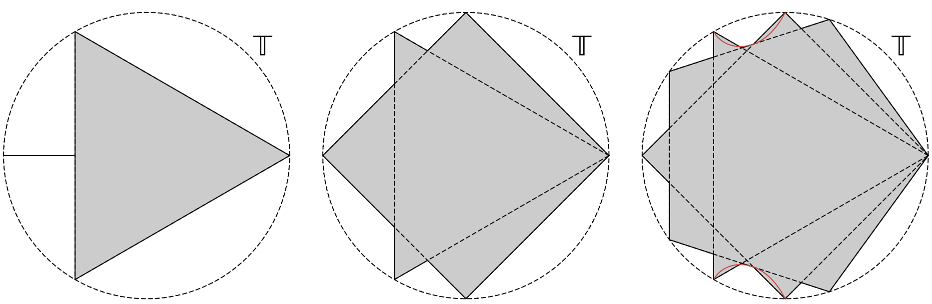
- For $n = 2, 3, 4$, it is the numbers in the union of the n smallest regular polygons in $\mathbb{D}$ (call it $\boldsymbol{PM_n}$).
- This is not the case for $n = 5$.
- For $n = 4$, **convexly independant** eigenvectors were considered in the proof.

2 The conjecture

- Eigenvalues of $n \times n$ bistochastic matrices with **conv. ind.** eigenvectors lie in PM_n.

3 Proof

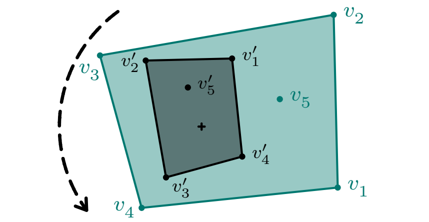
- A **convexly ind.** eigenpair $(re^{i\theta}, v)$ is **maximal** $\iff$ r is largest for θ among all $n \times n$ bistochastic matrices.
- If (λ, v) is **maximal**, then
  $$v' = \left(\frac{v_1+v_2}{2}, \frac{v_2+v_3}{2}, \dots, \frac{v_n+v_1}{2}\right)$$
  is also **maximal**.
- Repeated application of this transformation converges to n points on an **ellipse**.
- Maximizing r for all v on an ellipse yields our main result.



Extra facts & figures

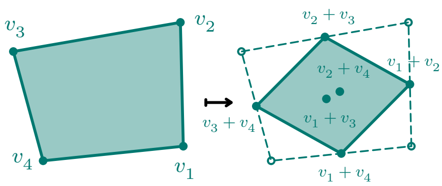
1) Stochastic matrices

Theorem. *(λ, v) is an eigenpair of a stochastic matrix* $\iff$ $\lambda\,\mathrm{conv}(v_j) \subseteq \mathrm{conv}(v_j)$.

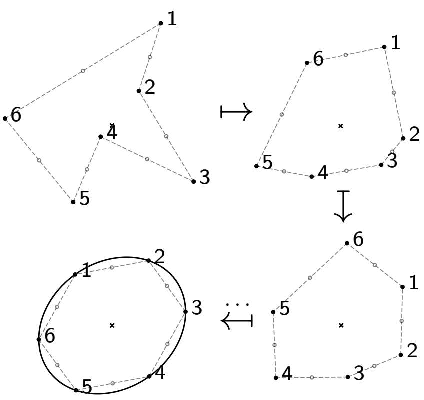


2) Bistochastic matrices

Theorem. *(λ, v) is an eigenpair of a bistochastic matrix* $\implies$ *For every* $1 \le k \le n$, $\lambda\,\mathrm{conv}(v_{j_1} + \dots + v_{j_k}) \subseteq \mathrm{conv}(v_{j_1} + \dots + v_{j_k})$.



3) Iterated averages



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