MR4913744 15B51 15A03 15A15 15A60 65F40

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The range of combined matrices and doubly quasi-stochastic matrices of order 3. (English. English summary)

Linear Algebra Appl. **721** (2025), 399–418.

Given a nonsingular matrix A, the combined matrix of A is the matrix $\mathcal{C}(A) = A \circ (A^t)^{-1}$, where $\circ$ denotes the entrywise product.

An $n \times n$ matrix U is said to be doubly stochastic if it has nonnegative entries and each of its rows and columns sums to one. If the requirement that all entries be nonnegative is dropped and only real entries are required, then such matrices are called doubly quasi-stochastic.

Since a characterization of the 3×3 real matrices whose combined matrix equals a given doubly stochastic matrix has recently been obtained (by a group that includes some of the present authors), it is natural to turn to the related problem of determining the conditions that the entries of a given doubly quasi-stochastic 3×3 matrix must satisfy in order for a real matrix to exist whose combined matrix is precisely that doubly quasi-stochastic matrix.

This question is settled in Section 2, where two cases are distinguished: first, the case in which the first row and first column of the matrix contain no zeros, and second, the case in which they do. Both results are supplemented with procedural diagrams to aid understanding, and an illustrative example is also provided.

Section 3 first shows that, given any three real numbers, one can always construct a 3×3 doubly quasi-stochastic matrix whose main diagonal consists exactly of those three numbers. With this preliminary result in hand, the section then turns to the following question: if instead of being given the entire doubly quasi-stochastic 3×3 matrix one is given only its main diagonal, does there exist a real matrix whose combined matrix has these three real numbers on its main diagonal? Conditions on the diagonal entries are obtained, and two illustrative examples conclude the section.

Section 4 examines the conditions under which the matrices obtained in the first objective are totally positive—that is, matrices whose minors of every order are positive. Once again, the results are accompanied by two illustrative examples.

Section 5 presents three algorithms used to obtain the numerical results from the preceding sections. Each algorithm is followed by a brief numerical example.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.