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More on distribution of eigenvalues of smooth Toeplitz matrices. (English) Zbl 08108412

Pure Appl. Funct. Anal. 10, No. 3, 593-604 (2025).

Given a sequence $\{a_j\}_{j=0}^{\infty}$ of non zero complex numbers, let A_{mn} denote the Toeplitz matrix $(a_{m-j+k})_{1\le j,k\le n}$, where $a_j = 0$ for $j < 0$. The author is interested in the distribution of the eigenvalues of A_{mn} as $n \to \infty$ and $m = m(n)$ satisfies $\frac{1}{R} \le \frac{m}{n} \le R$ for some $R > 1$, and under the basic assumption that the sequence $\{a_j\}_{j=0}^{\infty}$ is "smooth" in the sense that

$$\frac{a_{j-1}a_{j+1}}{a_j^2} \;=\; \exp\left(-\frac{1}{\rho_j}\big(1+o(1)\big)\right),$$

for so-called asymptotic comparison sequence $\{\rho_j\}_{j\ge 1}$.

In this article – which, as its title suggests, follows on from previous publications by the same author in the same field of research – we focus on counting measures that weight absolute values of the eigenvalues, or absolute values of their real parts. We also obtain an upper bound on the determinant of A_{mn}.

To conclude the article, three problems are formulated:

(1) Formulate conditions for $\det(A_{mn})$ to be non zero;

(2) Estimate below the eigenvalue of smallest modulus of A_{mn}, under suitable conditions;

(3) Formulate conditions that permit some form of asymptotics for $\det(A_{mn})$.

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MSC:

15B05 Toeplitz, Cauchy, and related matrices
15A18 Eigenvalues, singular values, and eigenvectors
15A15 Determinants, permanents, traces, other special matrix functions

Keywords:

Toeplitz matrices; eigenvalue distribution

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