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★**Logarithmic norms.**

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Gustaf Söderlind's *Logarithmic norms* is a rare kind of mathematical monograph: it takes a concept that is both underappreciated and highly versatile and elevates it to the status of a central methodological tool in applied mathematics. This book is both a rigorous theoretical treatment and a cross-disciplinary exploration of logarithmic norms as they appear in matrix theory, operator theory, nonlinear analysis, and differential equations. It succeeds in clarifying and systematizing scattered results (some quite recent), bringing coherence and depth to an often overlooked yet conceptually rich topic.

Söderlind opens his book with an interesting analogy: not mentioning the logarithmic norm in operator theory is akin to writing a textbook on complex analysis without ever referring to the real part of a complex number. This introduces a core theme of the book, namely that the logarithmic norm plays a role with respect to the operator norm, similar to that of the real part to the absolute value. Despite this, the logarithmic norm remains largely neglected in standard curricula and literature. Söderlind sets out to change this by offering an integrated and comprehensive treatment of the subject.

The book is organized into four parts, each composed of 6 to 8 chapters. These parts are not uniform in length however.

The (shorter) first part introduces the logarithmic norm from first principles, aiming for accessibility. It motivates the need for the logarithmic norm through simple examples from stability theory and presents connections with spectral theory and differential inequalities. Special attention is given to the fact that the logarithmic norm can be negative, an essential property for distinguishing stable from unstable systems.

The second part focuses on matrix theory, particularly within a Hilbert space setting. Here, the logarithmic norm is defined via inner products, and links are drawn with the numerical radius, the polar and Cartesian decompositions, and unitarily invariant norms like the Frobenius norm. The discussion includes results on positive definite matrices, spectral theory, Möbius transformations, and connections to classical inequalities such as the von Neumann inequality and the Kreiss matrix theorem. This section is dense with results.

In the third part, Söderlind generalizes the framework to Lipschitz continuous nonlinear maps. Classical concepts such as contractivity, coercivity, and monotonicity are recast using Lipschitz and logarithmic Lipschitz constants. Several nonlinear models are analyzed, with an emphasis on perturbation theory and stability. This part is particularly useful for readers interested in extending linear operator theory to nonlinear settings.

The (longer) final part investigates the application of logarithmic norms to differential operators, particularly in one-dimensional settings and two-point boundary value problems. Through the method of spatial discretization, Söderlind connects results from matrix theory to those from partial differential equations, showing how elliptic and parabolic problems can be handled using the same conceptual machinery.

Each part begins with a concise 'Orientation' chapter, followed by structured and formal development. While the style is clear and didactic, the book does assume a solid background in functional analysis, operator theory, differential equations, and matrix analysis. The target audience is thus *not* introductory-level students, but graduate students and researchers with prior training.

Among the book's many strengths is its unifying vision. Söderlind systematically develops the theory of logarithmic norms as a bridge between disparate areas of mathematics. While several works that have become classics cover the notions and concepts which underpin much of the theory in Söderlind's work, the reviewer knows of no other book covering logarithmic norms in quite the same way or replicating the unifying in-depth treatment found in *Logarithmic norms*.

Another merit is the author's emphasis on geometric intuition. Rather than relying solely on formal derivations, Söderlind offers geometric interpretations enhancing the reader's conceptual understanding and facilitating the application of abstract theory to concrete problems. While the book contains few historical passages, those that are present are insightful, accurate, and to the point.

The book's narrow focus is both its strength and its limitation. While it offers an exhaustive treatment of logarithmic norms, its specialized nature and lack of pedagogical features (such as exercises, problems or extended examples) make it less suitable for classroom use. Moreover, while some applications are touched upon, the examples are, to a large extent, theoretical rather than computational. The absence of computational examples may be a drawback for readers hoping for algorithmic or numerical guidance.

This book is an important contribution to applied mathematics. It is rigorous, well-written, and conceptually rich, offering a systematic treatment of a mathematical tool that deserves greater attention. It is best suited for researchers and practitioners in numerical analysis, matrix theory, or stability theory. *Frédéric Morneau-Guérin*