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Intertemporal Asset Pricing without Risk-Free Security, Zero-Beta Portfolio and Consumption Data

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Abstract

In this note, we propose an intertemporal asset pricing approach without risk-free security, zero-beta portfolio and consumption data. The framework is based on the consumption model, logarithmic utility function, and relative returns. Our main result indicates that the expected return of an asset is equal to the return of the market portfolio (the benchmark), plus a positive or negative risk adjustment, directly proportional to its *relative-beta* (obtained from the difference between the *usual market beta* and *one*). This offers an additional and easy-to-apply tool to estimate the required return of an asset and characterize the equilibrium risk-return relationship.

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Declarations

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1. Introduction

The main prediction of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) indicates that the expected return of an asset is equal to the risk-free return, plus a positive risk premium directly proportional to its standard beta. On this basis, many extension models have been proposed for asset pricing. For example, we have the zero-beta CAPM of Black (1972), the intertemporal CAPM of Merton (1973), the arbitrage pricing theory (APT) of Ross (1976), the consumption CAPM of Breeden (1979), the recursive preference model of Epstein and Zin (1989), the three-factor model of Fama and French (1993), the two-beta model of Campbell and Vuolteenaho (2004), and the three-beta model of Campbell et al. (2018). All of these models predict that the expected return on any risky asset (ER) equals the return of a risk-free asset (R_F), or the expected return of a zero-beta portfolio (ER_Z), plus a positive risk premia. This can be illustrated in the following manner:

$$ER = R_F \; (or \; ER_Z) + risk \; premia.$$

In this note, we propose a complementary method: an intertemporal asset pricing approach without risk-free security or zero-beta portfolio. Instead of beginning the estimation with the lowest return ($R_F \; or \; ER_Z$), we begin the estimation with the average return (on the market), and then add a positive or negative risk adjustment. More precisely, we propose to utilize the following adjustment:

$$ER = ER_M \pm \; risk \; adjustment,$$

where ER_M represents the expected return of the market portfolio, seen as a benchmark (or reference point).

Concerning the risk-free asset, our approach is motivated by the fact that the future is uncertain, and that all financial investments carry some degree of risk. Moreover, according to Black (1972, p. 446) and others, empirical results suggest that the risk-free assumption, related to the standard CAPM, does not hold.[1] Concerning the zero-beta portfolio, our approach is here motivated by the fact that this portfolio is unobservable and leads to considerable empirical difficulties, as recently pointed out by Beaulieu et al. (2013), Beaulieu et al. (2022), and Beaulieu et al. (2023). Besides, as in Campbell (1993), Campbell and Vuolteenaho (2004), and Campbell et al. (2018), we replace consumption in the standard intertemporal model. This time, our approach is motivated by the following points: first, consumption of asset-market participants may be poorly proxied by aggregate consumption (see Campbell, 1993, p. 488); second, the failure of the consumption-CAPM in practice (see Campbell and Cochrane, 2000, p. 2864), and third, the effect of measurement errors in available consumption data sets (see Kroencke, 2017, p. 47).

[1] See, for example, Fama and French (2004) or Campbell (2018, p. 66).

However, none of the abovementioned studies propose an asset pricing approach without risk-free asset, zero-beta portfolio and consumption data, in concert. Moreover, none of them propose to price assets using a risk adjustment to a benchmark.

Our framework is less complex than the Epstein-Zin preferences structure used by Campbell et al. (2018), for example. It is based on the consumption model with a logarithmic utility function. It also uses expected relative returns ($ER - ER_M$) instead of excess returns ($ER - R_F$). Our main result indicates that the expected return of an asset is equal to the market return (the benchmark), plus a positive or negative risk adjustment, directly proportional to its *relative-beta* (obtained from the difference between the *standard beta* and *one*). This suggests that in an intertemporal context, we can characterize the equilibrium risk-return relationship without the riskless asset, zero-beta portfolio and consumption data.

In light of this, the paper's main contributions can be summarized as follows. First, it innovates in suggesting that any asset expected return will correspond to the benchmark return after risk adjustment. Second, it proposes a direct pricing prediction without the (hypothetical) riskless asset, (unknown) zero-beta portfolio and (suspicious) consumption data (in concert). Third, it offers an additional and easy-to-apply tool for investors or managers to estimate the cost of equity capital and evaluate the performance of a fund. Fourth, as with any new theory, it provides new predictions for future empirical tests.

In Section 2, we develop our relative-beta model, for asset pricing. In Section 3, we propose two potential applications of our theoretical model. In Section 4, we summarize and conclude.

2. The relative-beta model

In this section, we progressively present our variables and parameters, before showing the familiar Euler equation with a logarithmic utility function. Thereafter, we integrate relative returns and express our main result in the form of a risk-return relationship.

Logarithmic utility function

In accordance with the consumption-based model, our intertemporal framework assumes a restrictive economy in which the (aggregate) representative investor maximizes the following time-separable utility function, given the available information at time t:[2]

$$U(C_t) + E_t \sum_{s=1}^{\infty} \delta^s U(\tilde{C}_{t+s}), \tag{1}$$

[2] See Cochrane (2005, Chapter 1).

subject to resource constraints, where δ is the time discount factor $(0 < \delta < 1)$, U is an increasing concave and derivable function, C_t denotes aggregate consumption at time t, and $\tilde{C}_{t+s}$ denotes aggregate consumption at time $t+s$ $(s = 1, 2, \dots, \infty)$.[3] Knowing that consumption comes from dividends, the result of this investor optimization problem reveals that the price of asset i, at time t, P_{it}, is such that $(i = 1, 2, \dots, N)$

$$P_{it} = E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \widetilde{D}_{i,t+s}\,, \tag{2}$$

where $\widetilde{D}_{i,t+s}$ is the dividend of asset i, at time $t+s$. Assuming a logarithmic utility function such that $U(C) = \ln(C)$, and $U'(C) = 1/C$, we have[4]

$$P_{it} = E_t \sum_{s=1}^{\infty} \delta^s \frac{C_t}{\tilde{C}_{t+s}} \widetilde{D}_{i,t+s}\,, \tag{3}$$

and for the aggregate market portfolio, we also have

$$P_{Mt} = E_t \sum_{s=1}^{\infty} \delta^s \frac{C_t}{\tilde{C}_{t+s}} \widetilde{D}_{M,t+s}\,, \tag{4}$$

where P_{Mt} corresponds to the aggregate price of the market portfolio at time t, and where $\widetilde{D}_{M,t+s}$ is the aggregate dividend of the market portfolio at time $t+s$. Because $\tilde{C}_{t+s}$ is equivalent to $\widetilde{D}_{M,t+s}$, equation (4) becomes

$$P_{Mt} = E_t \sum_{s=1}^{\infty} \delta^s C_t\,. \tag{5}$$

Equation (5) represents a perpetuity, and it is easy to prove that

$$P_{Mt} = C_t/a. \tag{6}$$

with $\delta \equiv 1/(1+a)$ and $0 < a < 1$. Therefore, given the available information at time t, we can write

[3] In this section, the tilde (~) indicates a random variable. Operators E_t, V_t, and COV_t refer respectively to mathematical expectations, variance and covariance, where index t implies that we consider the available information at time t.

[4] As noted by Campbell (2018, p. 284), in an intertemporal context, for a long-term investor with log utility, the appropriate measure of risk corresponds to the standard beta (with the market portfolio).

$$\tilde{P}_{M,t+1} = \tilde{C}_{t+1}/a, \tag{7}$$

where $\tilde{P}_{M,t+1}$ corresponds to the random price of the market portfolio, at time $t+1$. Recursively (with log utility), equation (3) implies the following two-time expression (see Cochrane, 2005, p. 27):

$$P_{it} = E_t\left[\delta \frac{C_t}{\tilde{C}_{t+1}}(\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})\right], \tag{8}$$

where $\tilde{P}_{i,t+1}$ is the random price of asset i at time $t+1$. Integrating equations (6) and (7) in (8), indicates

$$P_{it} = E_t\left[\frac{1}{(1+a)}\,\frac{a\,P_{Mt}}{a\,\tilde{P}_{M,t+1}}(\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})\right], \tag{9}$$

or, equivalently

$$P_{it} = E_t\left[\frac{a\,P_{Mt}}{a\,\tilde{P}_{M,t+1} + a\,\tilde{C}_{t+1}}(\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})\right]. \tag{10}$$

After simplifications, we can express

$$P_{it} = E_t\left[\frac{P_{Mt}}{\tilde{P}_{M,t+1} + \tilde{C}_{t+1}}(\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})\right], \tag{11}$$

which is equivalent to

$$P_{it} = E_t\left[\tilde{R}_{M,t+1}^{-1}(\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})\right], \tag{12}$$

where $\tilde{R}_{M,t+1} \equiv (\tilde{P}_{M,t+1} + \tilde{D}_{M,t+1})/\tilde{P}_{Mt}$ represents the return of the market portfolio between time t and $t+1$. Dividing by P_{it} on each side of equation (12), we obtain the following result:

$$1 = E_t\left[\tilde{R}_{M,t+1}^{-1}\tilde{R}_{i,t+1}\right], \tag{13}$$

where $\tilde{R}_{i,t+1} \equiv (\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1})/\tilde{P}_{it}$ is the return of asset i, between time t and $t+1$. Here, equation (13) represents a particular formulation of the familiar Euler equation.

Relative returns

In the same manner, for the market portfolio, we have

$$1 = E_t\left[\tilde{R}_{M,t+1}^{-1}\tilde{R}_{M,t+1}\right], \tag{14}$$

and equation (13) minus (14) indicates that

$$0 = E_t\left[\tilde{R}_{M,t+1}^{-1}(\tilde{R}_{i,t+1} - \tilde{R}_{M,t+1})\right], \tag{15}$$

or, to simplify the notation

$$0 = E_t\left[\tilde{R}_{M,t+1}^{-1}\tilde{R}_{i,t+1}^{r}\right], \tag{16}$$

where $\tilde{R}_{i,t+1}^{r} \equiv \tilde{R}_{i,t+1} - \tilde{R}_{M,t+1}$ represents the *relative return* of asset i, between time t and $t+1$. This value is frequently used in portfolio performance management. It serves to analyze the performance of an asset (or portfolio) compared to the market (or benchmark). From equation (16), the basic covariance definition implies that

$$COV_t\left[\tilde{R}_{M,t+1}^{-1}, \tilde{R}_{i,t+1}^{r}\right] = -E_t\left[\tilde{R}_{M,t+1}^{-1}\right]E_t\left[\tilde{R}_{i,t+1}^{r}\right], \tag{17}$$

and isolating the asset expected relative return, we get

$$E_t\left[\tilde{R}_{i,t+1}^{r}\right] = -COV_t\left[\tilde{R}_{M,t+1}^{-1}, \tilde{R}_{i,t+1}^{r}\right]/E_t\left[\tilde{R}_{M,t+1}^{-1}\right]. \tag{18}$$

Taking a Taylor approximation around the point $\mu_{Mt} \equiv E_t\left[\tilde{R}_{M,t+1}\right]$, allows us to write the following approximation:

$$E_t\left[\tilde{R}_{i,t+1}^{r}\right] \approx -COV_t\left[\mu_{Mt}^{-1} - \mu_{Mt}^{-2}(\tilde{R}_{M,t+1} - \mu_{Mt}), \tilde{R}_{i,t+1}^{r}\right]/E_t\left[\tilde{R}_{M,t+1}^{-1}\right], \tag{19}$$

and using basic covariance properties, equation (19) becomes

$$E_t\left[\tilde{R}_{i,t+1}^{r}\right] \approx \mu_{Mt}^{-2}COV_t\left[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}^{r}\right]/E_t\left[\tilde{R}_{M,t+1}^{-1}\right]. \tag{20}$$

Ignoring the approximation and integrating the definition of the relative returns gives

$$E_t\left[\tilde{R}_{i,t+1}^{r}\right] = \mu_{Mt}^{-2}COV_t\left[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1} - \tilde{R}_{M,t+1}\right]/E_t\left[\tilde{R}_{M,t+1}^{-1}\right]. \tag{21}$$

Again, using basic covariance properties, we have

$$E_t\left[\tilde{R}_{i,t+1}^{r}\right] = \mu_{Mt}^{-2}\left(COV_t\left[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}\right] - V_t\left[\tilde{R}_{M,t+1}\right]\right)/E_t\left[\tilde{R}_{M,t+1}^{-1}\right]. \tag{22}$$

Multiplying by $V_t[\tilde{R}_{M,t+1}]$ on each side of equation (22) indicates, after simple manipulations

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{M,t+1}] + \lambda_t(\beta_{it} - 1), \tag{23}$$

$$\lambda_t \equiv \mu_{Mt}^{-2} V_t[\tilde{R}_{M,t+1}] / E_t[\tilde{R}_{M,t+1}^{-1}],$$

$$\beta_{it} \equiv COV_t[\tilde{R}_{M,t+1}, \tilde{R}_{i,t+1}] / V_t[\tilde{R}_{M,t+1}],$$

where β_{it} corresponds to the *standard beta* of asset i, at time t, Finally, we obtain this linear relationship

$$E_t[\tilde{R}_{i,t+1}] = E_t[\tilde{R}_{M,t+1}] + \lambda_t \beta_{it}^r, \tag{24}$$

with $\beta_{it}^r \equiv (\beta_{it} - 1)$.

We term the coefficient β_{it}^r, the *relative-beta* of asset i at time t, knowing that the *standard beta* of the benchmark corresponds to *one* (by definition). It measures the asset's return sensitivity to the market factor, from a relative point of view. A particular relative-beta higher (lower) than zero indicates that the asset's sensitivity is higher (lower) than the average.

Equation (24) represents our main result. It demonstrates, firstly, without risk-free asset or zero-beta portfolio, that the expected return of an asset is linearly related to its relative-beta. In equation (24), the asset's expected return is positively related to its relative-beta, since parameter λ_t is greater than zero. Indeed, knowing that μ_{Mt}^{-2}, $V_t[\tilde{R}_{M,t+1}]$ and $E_t[\tilde{R}_{M,t+1}^{-1}]$ are all mathematically positive, then, by construction, parameter λ_t is also positive. In accordance with the usual *risk averse postulate*, this implies that a relative-beta represents an appropriate risk measure of an asset. This implies, in addition, that parameter λ_t gives an appropriate risk price. Therefore, equation (24) demonstrates that the expected return of an asset is equal to the benchmark return, plus a positive or negative risk adjustment, directly proportional to its relative-beta. If the asset's relative-beta is superior (inferior) to zero, then its risk adjustment ($\lambda_t \beta_{it}^r$) is positive (negative), and its required return is higher (lower) than the average.

Overall, our above developments reveal that a theoretical description of the risk-return relationship can be obtained, in an intertemporal context, without using two restrictive assumptions related to the risk-free asset or zero-beta portfolio. Moreover, it is easy to prove that the standard CAPM and zero-beta CAPM represent two particular cases of our main result (see Appendix A).

3. Possible applications

We suggest two possible applications for our theoretical model that apply the current approach to reality. More precisely, we can utilize our model to (1) estimate the cost of equity capital, and (2) evaluate the realized performance of a stock or portfolio.

The cost of equity capital

As noted by Fama and French (2004, p. 43), "... finance textbooks often recommend using the Sharpe-Lintner CAPM risk-return relation to estimate the cost of equity capital." The prescription is to determine the risk-free rate, estimate the stock's market beta, and combine these values with the average market price of risk to approximate the firm's cost of equity. Here, we propose using our main result to obtain a similar estimation with no risk-free rate (or zero-beta portfolio return). Our prescription, this time, is to determine the average market return, estimate the stock's market beta as well as the corresponding relative-beta, and combine these values with the average market price of risk to approximate the firm's cost of equity. More precisely, the approximate cost of equity of firm i, k_i, will be such that

$$k_i = \bar{R}_M + \bar{\lambda}\hat{\beta}_i^r, \tag{29}$$

where $\bar{R}_M$ is the average market return, $\bar{\lambda}$ is the average market price of risk, and $\hat{\beta}_i^r$ is the estimated relative-beta of firm i, given by the corresponding estimated beta ($\hat{\beta}_i$). In practice, this procedure simplifies the estimation of a firm's cost of equity since the only macroeconomic variable to be selected and calculated is the market index return, with no estimation required for the risk-free rate or zero-beta portfolio return. The risk-free rate is supposed to be constant, but, in reality, it fluctuates. Its determination thus implies an additional source of errors for firm managers or investors. Moreover, these potential errors are greater for the zero-beta portfolio, since this type of fund does not exist in financial markets. To simplify the estimation, we can also assume that $E_t\left[\tilde{R}_{M,t+1}^{-1}\right]$ is approximately equal to $E_t[\tilde{R}_{M,t+1}]^{-1}$ which implies that: $\lambda_t = V_t\left[\tilde{R}_{M,t+1}\right]E_t[\tilde{R}_{M,t+1}]^{-1}$. At the end of 2023, the average return (1993-2023) of the S&P 500 Index was estimated to be near 10%, with a standard error of 18%. Thus, using the S&P 500 index as a market proxy, we can propose that the average market price should be equal to: $\bar{\lambda} = (0.18)^2 \text{ x } (1.10)^{-1} = 0.03$. In this manner, the estimated cost of equity of a firm, on the US market, will be

$$k_i = 0.10 + (0.03)\hat{\beta}_i^r.$$

To connect this to real-world settings, consider the following concrete example. In 2024, the stock beta of the Ford Motor Company (F)[5] was estimated to be 1.61, which

[5] See Yahoo Finance.

implies a corresponding relative-beta of 0.61. Therefore, using our previous empirical data, we can establish that the estimated cost of equity for the Ford Motor Company (k_F) is such that

$$k_F = 0.10 + (0.03)0.61,$$

and is equal to 11.83%. This method can easily be reproduced for many other companies in the stock market.

The performance of portfolios and individual stocks

Additionally, as noted by Fama and French (2004, p. 44), "The CAPM is also often used to measure the performance of mutual funds and other managed portfolios." For stock portfolios, for example, one possible approach is to calculate the observed return of the portfolio P (R_P) and compare this value to the corresponding value predicted by the CAPM. Here, we propose to compare the portfolio return with the expected value predicted by our model as shown below:

$$R_P^* = R_M + \bar{\lambda}\hat{\beta}_P^r, \tag{30}$$

where R_P^* is the normal return of portfolio P, R_M is the observed market return, and $\hat{\beta}_P^r$ is the estimated relative-beta of portfolio P. If the observed return R_P is superior (inferior) to R_P^*, then the portfolio presents positive (negative) abnormal returns ($R_P - R_P^*$), and the portfolio's performance is considered to be *good* (*poor*). This procedure will simplify the estimation of the normal return, since, as before, the only macroeconomic variable to be selected and calculated is the market index return, which eliminates the potential errors associated with the choice of the risk-free rate (or the zero-beta portfolio return). This procedure will also eliminate the integration of a particular security (such as a T-bill) that is not in the same security category as the stock portfolio. Moreover, He, O'Connor and Thijssen (2022) clearly demonstrated that there is no consistent asset across countries (such as gold, T-bills or bonds) that can be considered with no systematic risk and thus serve as a proxy for the lowest-risk rate in the CAPM. In that sense, *in a real-world setting*, our approach can help to avoid the following difficulties: (1) choosing the type of proxy we want for the lowest-risk asset (for example, gold, T-bills or bonds); (2) determining the appropriate term for the selected T-bills or bonds; and (3) estimating the average empirical rate of the selected riskless asset.

As for March 2025, the S&P 500 has delivered an observed annual return of 14.27%, over the past 5 years, while the corresponding return for the Ford Motor Company was 9.78%. Hence, by adapting the previous development to an individual stock (a portfolio of only one asset), we can easily deduce that the normal annual return for stock F (R_{Ford}^*) over the last 5 years is given by

$$R^*_{Ford} = R_M + (0.03)0.61 = 0.1427 + (0.03)0.61 = 16.10\%, \quad (30)$$

and that the observed return of the Ford Motor Company (9.78%) was inferior to its normal return (16.10%).

4. Conclusion

Employing a simple intertemporal framework, this paper demonstrates that the expected return of an asset is equal to the market portfolio return, plus a positive or negative *risk adjustment* directly proportional to its *relative-beta*. This suggests that an equilibrium risk-return relationship can be obtained without the riskless asset, zero-beta portfolio and consumption data.

In the construction of our model, we used a restrictive logarithmic utility function. For a future model, it might be interesting to use a less restrictive function such as the standard power utility function. Furthermore, future research could build an empirical test of our relative-beta model.[6]

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[6] In Appendix B, we present an exploratory application of our model that could be helpful for future empirical research.

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Appendix A: Particular cases

It is easy to prove that the standard CAPM and the zero-beta CAPM represent two particular cases of our main result. In fact, for the zero-beta portfolio, identified by the letter *Z*, our equation (24) predicts that

$$E_t\left[\tilde{R}_{z,t+1}\right] = E_t\left[\tilde{R}_{M,t+1}\right] + \lambda_t(0-1) = E_t\left[\tilde{R}_{M,t+1}\right] - \lambda_t, \tag{A1}$$

where $\tilde{R}_{z,t+1}$ represents the return of the zero-beta portfolio at time $t+1$. Thus, equation (24) minus (A1) shows that

$$E_t\left[\tilde{R}_{i,t+1}\right] - E_t\left[\tilde{R}_{z,t+1}\right] = \lambda_t + \lambda_t(\beta_{it} - 1) = \lambda_t\beta_{it}, \tag{A2}$$

and integrating equation (A1) in (A2), we get

$$E_t\left[\tilde{R}_{i,t+1}\right] = E_t\left[\tilde{R}_{z,t+1}\right] + (E_t\left[\tilde{R}_{M,t+1}\right] - E_t\left[\tilde{R}_{z,t+1}\right])\beta_{it}, \tag{A3}$$

which is equivalent to the zero-beta CAPM (in its conditional form). In the same manner, reproducing the above development, we can write

$$E_t\left[\tilde{R}_{i,t+1}\right] = R_{F+1} + (E_t\left[\tilde{R}_{M,t+1}\right] - R_{F+1})\beta_{it}, \tag{A4}$$

where R_{F+1} represents the return of the riskless asset at time $t+1$, which is now equivalent to the standard CAPM. Therefore, equations (A1) to (A4) reveal that the crucial predictions of the standard CAPM and zero-beta CAPM can be viewed as two particular cases of our main result, if we accept the following restrictive assumptions: (1) *a (pure) risk-free asset exists*, and (2) *the representative investor can identify a risky asset (or portfolio) that is uncorrelated with the market index.*

Appendix B: Future empirical research

In this Appendix B, we present an exploratory application of our model that could be helpful for future empirical research.

Ignoring index *t* in operators E_t, V_t, and COV_t allows us to express our main prediction, expressed by equation (24), in the following manner:

$$E\left[\tilde{R}_{i,t+1}\right] = \lambda_0 + \lambda_1\beta_i^r, \tag{B1}$$

with $\lambda_0 \equiv E\left[\tilde{R}_{M,t+1}\right]$ and $\lambda_1 \equiv \mu_M^{-2} V\left[\tilde{R}_{M,t+1}\right] / E\left[\tilde{R}_{M,t+1}^{-1}\right]$.

Given that, a potential empirical test of our model can (firstly) be based on the following three implications of the relation between expected return and relative-beta. First, expected returns on all assets are linearly related to their relative-betas. Second, the intercept (λ_0) of the line equals the expected market return. Third, the slope of the line (λ_1) is positive and increases as market variability increases. Following the major empirical studies on the CAPM, a simple investigation of our model could employ a cross-sectional regression to test the above implications. Here, the approach would consist of regressing a cross-section of average asset returns on historical asset relative-betas. The model will predict that the intercept in this regression is the average market return, while the corresponding slope is the average market price.

To simplify the empirical investigations, we can, as before, assume that $E\left[\tilde{R}_{M,t+1}^{-1}\right]$ is approximately equal to $E[\tilde{R}_{M,t+1}]^{-1}$, which implies that: $\lambda_1 = V\left[\tilde{R}_{M,t+1}\right]E[\tilde{R}_{M,t+1}]^{-1}$. Using our previous observations, a cross-sectional test can then be adapted to verify different hypotheses similar to the propositions below:

H1: The estimate of λ_0 = 10%;

H2: The estimate of λ_1 = 3%.

As noted by Fama and French (2004) and Campbell (2018), most of the major empirical tests of the standard CAPM reveals that the relation between beta and average return is much flatter than the standard CAPM predicts. The observations indicate that the intercept is higher than the historical risk-free rate ($\bar{R}_F$) estimated with T-bills, and the slope is lower than the historical excess market return ($\bar{R}_M - \bar{R}_F$). This is not inconsistent with the zero-beta CAPM, which predicts only that the beta premium (or market price) is positive. However, as we have already noted, the return of the zero-beta portfolio is unknown, as are the historical zero-beta return ($\bar{R}_Z$) and the corresponding historical market price ($\bar{R}_M - \bar{R}_Z$). Therefore, it is difficult to determine if the intercept and the slope of the empirical cross-sectional regression have reasonable values. A future empirical test of our model based on the above procedure and hypothesis will not present this specific difficulty. Moreover, we can reasonably expect that the slope in this eventual cross-sectional regression will also be low and in line with our historical average risk price (here estimated to be near 3%).