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Louis Gérard's thesis on non-Euclidean geometry (1892): a new step in the history of this discipline. (La thèse de Louis Gérard sur la géométrie non euclidienne (1892) : une nouvelle étape dans l'histoire de cette discipline.) (French. English summary) Zbl 07988984

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If the names of Nikolai Lobachevsky and János Bolyai are now inseparable from the challenge to Euclidean geometry, their works, which initiated this paradigm shift, went almost unnoticed when published in the 1820s and 1830s. It was not until the 1860s that their work was rediscovered and began to generate significant interest in the geometric theory based on the negation of the parallel postulate.

The interpretation of non-Euclidean geometry within known theories (differential geometry and projective geometry) by Eugenio Beltrami [1868] and later Felix Klein [1871] contributed to its acceptance by the mathematical community. However, philosophical debates on the subject continued until the end of the 19th century.

It was in this context that the French mathematician Louis Gérard defended, in 1892 at the Faculty of Sciences in Paris, a thesis titled Sur la géométrie non euclidienne [On Non-Euclidean Geometry]. Gérard's thesis – the most significant work specifically on this subject published in French-speaking countries since the Memoirs of Joseph-Marie De Tilly [1870] and Camille Flye Sainte-Marie [1871] – has not yet been the subject of an in-depth study, and this gap is what the article under review seeks to address.

First, a word about Louis Gérard. Born in 1859 in Grand, in the historical region of Lorraine, in eastern France, Gérard spent his entire career in secondary education. His thesis on non-Euclidean geometry thus represents his main contribution to mathematical research. The date of his death is unknown but is after 1939.

As this article demonstrates, Gérard's thesis serves as a transitional work between the contributions of the inventors of non-Euclidean geometry and the purely axiomatic research of the late 19th and early 20th centuries. For this reason, it deserves our attention.

Although trained in the early 1880s (before Weierstrass's methods became widespread in France), Gérard became familiar with these methods and avoided using infinitesimals in an intuitive manner. His thesis therefore reflects a heightened standard of rigor. In this regard, comparisons with earlier works are significant.

The author argues that Gérard was the first to propose a new approach to establishing non-Euclidean trigonometry by reasoning in an almost exclusively geometric manner and within the plane only. A decade later in Germany, Schur, Hilbert, and Liebmann would follow in his footsteps.

The second part of the thesis, as the author shows, is primarily dedicated to studying constructions using ruler and compass in non-Euclidean geometry. Gérard's interest in such problems may have stemmed from his reading of Bolyai's writings as well as his experience in secondary education. Using an original method, he arrives at results already established by Beltrami and Klein. The beginning of this second part, considered of lesser interest, is briefly summarized so that the article can then present several of its numerous applications to construction problems. These applications constitute the primary interest of this section. Since some of the reasoning in this part of the thesis is incomplete and often unclear, the author explains how to supplement them. Finally, the third part of Gérard's thesis provides a new contribution to the theory of polygonal areas, which the author contextualizes and puts into perspective.

The article also discusses the reception of Louis Gérard's thesis. It highlights key points from a report by Poincaré. While Poincaré clearly identifies the characteristics of Gérard's method – namely, a return to a synthetic approach similar to Euclid's – he seems largely uninterested in works that seek to reconstruct a known theory through a different approach and with stronger foundations. Other evaluations, also cited in the article, show that Gérard's approach attracted relatively little interest, particularly in France, where his work remained isolated. However, it is noted that Gérard's thesis deals in an often original way with a wide range of questions that resonated more strongly at the time in Italy and Germany. These research efforts would bear fruit and ultimately find their culmination in Hilbert's Grundlagen der Geometrie

[1899].

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