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On geometric circulant matrices with geometric sequence. (English) Zbl 1546.15016
Linear Multilinear Algebra 72, No. 10, 1555-1580 (2024).

After an introduction where a review of the literature on k-circulant matrices is given, the author presents and motivates the definition of a related class of matrices defined by *C. Kızılateş* and *N. Tuglu* [J. Inequal. Appl. 2016, Paper No. 312, 15 p. (2016; Zbl 1349.15060)], namely *geometric circulant matrices*, i.e., matrices completely determined by a nonzero complex number $k \in \mathbb{C}\backslash\{0\}$ and their first row in the sense that they are of the following form:

$$\operatorname{circ}_n\{_{k^*}(c_0, c_1, c_2, \dots, c_{n-1})\} = \begin{pmatrix} c_0 & c_1 & c_2 & \dots & c_{n-2} & c_{n-1} \\ kc_{n-1} & c_0 & c_1 & \dots & c_{n-3} & c_{n-2} \\ k^2c_{n-2} & kc_{n-1} & c_0 & \dots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k^{n-2}c_2 & k^{n-3}c_3 & k^{n-4}c_4 & \dots & c_0 & c_1 \\ k^{n-1}c_1 & k^{n-2}c_2 & k^{n-3}c_3 & \dots & kc_{n-1} & c_0 \end{pmatrix}.$$

More specifically, the paper examines a special case of geometric circulant matrices, where the entries in the first row form a geometric progression of common ratio $q \in \mathbb{R}\backslash\{0\}$ and of scale factor $g \in \mathbb{C}\backslash\{0\}$. In other words, the author considers matrices of the form

$$\operatorname{circ}_n\{_{k^*}(g, gq, gq^2, \dots, gq^{n-1})\},$$

and studies their determinant, Frobenius norm, as well as bounds for the spectral norm (i.e., the matrix norm induced by the ℓ_2-norm for vectors).

The main original results obtained and proved in this article generally come in pairs:

- An explicit formula (depending on k, n and q) is obtained for $\det(Q)$, where

  $$Q = \operatorname{circ}_n\{_{k^*}(1, q, q^2, \dots, q^{n-1})\}.$$

  This formula is then used to obtain an analogous formula for $\det(G)$, where

  $$G = \operatorname{circ}_n\{_{k^*}(g, gq, gq^2, \dots, gq^{n-1})\}.$$

- The Moore-Penrose inverse of

  $$\operatorname{circ}_n\{_{(\frac{1}{q^n})^*}(1, q, q^2, \dots, q^{n-1})\}$$

  is obtained for arbitrary natural number $n > 1$. This serves as a springboard for the computation of the Moore-Penrose of

  $$\operatorname{circ}_n\{_{(\frac{1}{q^n})^*}(g, gq, gq^2, \dots, gq^{n-1})\}$$

  for arbitrary natural number $n > 1$.

- The Moore-Penrose inverse of

  $$\operatorname{circ}_n\{_{(-1)^*}(1, -1, 1, \dots, -1, 1)\}$$

  is obtained for arbitrary odd natural number $n > 1$. Once again, this serves as a springboard for the computation of the Moore-Penrose of

  $$\operatorname{circ}_n\{_{(-1)^*}(g, -g, g, \dots, -g, g)\}$$

  for arbitrary odd natural number $n > 1$.

Lastly, and perhaps most interestingly, explicit formulas for the Frobenius norm of

$$\mathrm{circ}_n\{_{k^*}(g, gq, gq^2, \ldots, gq^{n-1})\}$$

are derived, along with upper and lower bounds for their spectral norm.

It is worth highlighting that the article contains a number of numerical examples to aid understanding.

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MSC:

15B05 Toeplitz, Cauchy, and related matrices
15A09 Theory of matrix inversion and generalized inverses
15A15 Determinants, permanents, traces, other special matrix functions
15A60 Norms of matrices, numerical range, applications of functional analysis to matrix theory
11C20 Matrices, determinants in number theory

Keywords:

geometric circulant matrix; geometric sequence; determinant of a matrix; norms of a matrix; generalized inverses of a matrix

Full Text: DOI

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