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Reviewed

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Syllogistic logic and mathematical proof.

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Review

Aristotle's foundation of logic as a discipline, codified in the *Organon*, and the development of proof in mathematics, embodied in Euclid's *Elements* and many later works, are landmark events in the history of human thought and, a fortiori, they are undoubtedly among the most remarkable intellectual achievements to emerge from Greece in Antiquity.

Curiously, it is hard to find in ancient writings any evidence of a sustained relationship between logical demonstration and mathematical proof. It is as if these two traditions developed on relatively autonomous bases. Indeed, it turns out that Greek logical texts (and, in their wake, medieval contributions) pay virtually no attention to mathematical proof as such. This is all the more surprising given that the *Organon* contains a number of programmatic statements which, on the face of it, indicate a thorough analysis of mathematical proofs, if not on the part of Aristotle, then at least on the part of his contemporaries. However, when mathematical examples are presented, they are mostly used to discuss specific issues that do not challenge Aristotle's strongly held general assumption that all valid types of argument, including mathematical proofs, are, or can be, rendered syllogistically.

In his writings, Aristotle adopted a somewhat hypocritical attitude towards the syllogism: on the one hand, he maintained a fairly liberal idea of what a syllogistic deduction is, while on the other hand, he asserted that any kind of mathematical theorem could be proved by means of a **BARbArA** syllogism (this syllogism must be understood as having two affirmative and universal premises and a conclusion, as in the well-known *Socrates is a man, all men are mortal, therefore Socrates is mortal*).

This dual attitude to the syllogism is reflected in the history of logic in the Western tradition, no doubt in large part due to the long, but subtle, persistence of Aristotle's conceptual hold on the field of logic until the 16th century. Although the theory of the syllogism and, more generally, the whole of logic, became an articulate and deeply developed discipline during the Middle Ages, most logicians and philosophers showed no particular interest in mathematical proofs until the second half of the 19th century, and maintained that the traditional syllogism of Aristotelian origin was the main tool for proving mathematical theorems. Only a few of them clearly expressed in their writings a sort of unease with the syllogism, and these assertions were most often followed by proposals to extend the classical Aristotelian syllogism in order, not

without some eagerness, to better re-establish the dogma that had been shattered. For their part, the mathematicians of the same period were not concerned with the traditional logical doctrine of Aristotelian origin, and they did not hesitate to use complex arguments that had no canonical Aristotelian syllogistic form in their proofs.

The reason why, for quite a long time, no one raised the problem of the adequacy of the logical apparatus to account for proofs in mathematics seems to lie mainly in the effective separation of the two disciplines. In the Western tradition, since Antiquity, logic and mathematics developed as two completely independent disciplines. Indeed, following the Boethian division of knowledge, logic (considered primarily as a science of language) was taught in what was known as the trivium, while mathematics (arithmetic and geometry) was taught in the quadrivium.

Nowhere is the dejection at the isolation that exists between these two solitudes expressed more melancholically (with well-felt and easily perceptible hints of frustration) than in an extract from a text by the British mathematician and logician Augustus De Morgan, which appeared in *The Athenaeum* on 18 June 1868 and was reproduced as an epigraph by the authors of the work under review: "We know that mathematicians care no more for logic than logicians for mathematics. The two eyes of exact sciences are mathematics and logic: the mathematical sect puts out the logical eye, the logical sect puts out the mathematical eye; each believing that it sees better with one eye than with two. The consequences are ludicrous".

In this captivating, well-documented monograph, philosophers Paolo Mancosu and Massimo Mugnai attempt to reconstruct "the vicissitudes of a conviction that for centuries prevailed among logicians and philosophers of Western culture" (p. 1), namely the belief that all the theorems of Euclid's *Elements* and, more generally, all mathematical theorems can be proved by means of the traditional Aristotelian syllogism.

Why did no one in the Greek tradition engage in detailed syllogistic reconstructions of mathematical proofs, despite Aristotle's assertion that all such proofs were reducible to syllogisms? How could generations of logicians assert that the syllogism is the universal canon of valid deduction, when they weren't sure that it was sufficient even for Euclid's first theorem? Or, if they knew that there were difficulties in doing so, why didn't they understand the importance of this fact and pursue the matter further? Why, before the 17th century, in Antiquity and the Middle Ages, independently of any contact with mathematics, did philosophers and logicians fail to realise the importance of relations and relational sentences in ordinary discourse as well as in theological and philosophical discourse and, above all, in arguments presented in mathematics, where they play an extremely important role? Without providing a complete and definitive answer to these questions, the authors manage to present clearly and intelligibly the main positions characterising this debate, and their interpretations provide at least a partial and convincing answer.

The study is divided into eight chapters, followed by a conclusion. Chapter 1 takes a close look at the relationship between logic and mathematics in ancient and medieval times. Chapter 2 presents the treatment of deductions containing oblique terms by William of Ockham (1285–1347) and John Buridan (1301–1358). Chapter 3 discusses the emergence of a more meticulous analysis of mathematical proofs (mainly geometrical) using the tools of syllogistic logic during the Renaissance, notably by Alessandro Piccolomini (1508–1578). Chapter 4 returns to the problem of oblique deductions and analyses the treatment given to them by several authors active in the 17th century. Chapter 5 is devoted to pre-Kantian philosophy in Germany, in particular Andreas Rüdiger (1673–1731) and Christian Wolff (1679–1754). Chapter 6 looks in greater depth at Kant's thought specifically in relation to the subject of interest. Chapter 7 is devoted to the work of Bernard Bolzano (1781–1848) and, finally, Chapter 8 focusses on the rich oeuvre of Augustus De Morgan (1806–1871).

Although the choice of thinkers whose thought is explored in this book may initially be somewhat surprising, it all makes sense once the criterion for inclusion is examined: all the authors who have contributed to the debate on whether mathematical proofs can be syllogised are covered, with the emphasis on those who have innovated conceptually, and less space given to those (including many well-known thinkers who occupy a prominent place in the history and

philosophy of mathematics) who have simply repeated points that were already made by other researchers before them.

In particular, the authors show us that the first embryonic forms of a logic of relations were developed during the 14th century by philosophers such as William of Ockham and John Buridan as part of their discussion of deductions in which so-called oblique terms (*terminus obliquus*) appear. The distinction between these oblique terms and the so-called straight terms (*terminus rectus*) was grammatical and had its origins in the work of the Latin grammarians of Antiquity. Medieval logicians knew that oblique terms implied a reference to relations and tried to develop a treatment of oblique terms within the framework of the traditional syllogism. However, given their lack of interest in mathematics, they did not associate oblique inferences with the inferences usually made by mathematicians when proving theorems. It was not until the work of Joachim Jungius (1587–1657) and Johannes Vegetius (1633–1691) in the 17th century that it became as obvious as it was undeniable, as the two authors point out, that the oblique deductions needed to prove mathematical theorems are, for the most part, not reducible to syllogisms. From the 17th century onwards, mathematics and mathematical symbolism began to take over the field of logic, albeit slowly, culminating in the work of George Boole (1815–1864) and Gottlob Frege (1848–1925).

In the midst of this effervescence, the question of inferences containing relations resurfaced with some force in the second half of the 19th century in the United Kingdom. The Scottish philosopher Thomas Reid (1710–1796) made it clear that certain relational inferences could not be reduced to syllogisms and that the traditional syllogistic doctrine was unsuitable for representing mathematical proofs. Reid's theses triggered a discussion in which Augustus De Morgan was involved. De Morgan was one of the last logicians in the tradition of Western logic to attempt to transform a mathematical (geometrical) demonstration into a chain of syllogisms, and he clearly recognised that the traditional Aristotelian syllogism was not suited to the task. However, he argued that, properly improved, the syllogism can still be a basic tool for proving mathematical theorems, the improvement consisting in looking at the syllogism from the aspect of the combination of relations. Thus, his particular views on the syllogism led De Morgan to turn his attention to the general concept of relations and to fully recognise their importance. Moreover, he took the first substantial step towards the construction of a logic of relations, a theory that was later developed by Charles Sanders Peirce (1839–1914) and Ernst Schröder (1841–1902). Having been unable to find, after De Morgan's successful attempts at the logic of relations, any other significant attempts to syllogise geometric theorems, the authors saw this as a natural stopping point for their account and study. However, they do not overlook the fact that since De Morgan's work—thanks in particular to Charles Sanders Peirce, Ernst Schröder, Gottlob Frege, Giuseppe Peano and Bertrand Russell—the syllogism has lost some of its special appeal to logicians and philosophers and has been relegated to the status of a small part of a more general theory of logic.

Mancosu and Mugnai take great care to point out that we must beware of making the mistake of believing that as soon as an appropriate theory of quantification and relational inferences was developed by mathematical logicians, this immediately led to a general recognition of the inadequacy of syllogisms to account for the logic of mathematical reasoning. They show us, with examples (notably the intriguing case of the *Allgemeine Erkenntnislehre* by the German philosopher Moritz Schlick, the leading thinker of the Vienna Circle), that many works of logic published in the second half of the 19th century and the first half of the 20th continued to present arguments defending the idea that all scientific knowledge, and mathematics in particular, can be presented syllogistically.

Ultimately, the book under review represents a remarkable work of scholarship. Written in prose that is at once clear, accessible, precise and lively, this monograph—given the broad scope of the subject without sacrificing any depth of analysis—is second to none. Readers will particularly appreciate the large number of quotations from lesser-known ancient authors, set in a context that reveals their meaning and followed by well-crafted and convincing interpretations and analyses.

Throughout the book, the authors demonstrate their in-depth knowledge and impressive mastery of both an abundance of primary sources (including a good number of well-known authors such as Aristotle, Avicenna, Bernard Bolzano,

Immanuel Kant, Gottfried Wilhelm Leibniz, Augustus De Morgan and John Stuart Mill), as well as a number of lesser-known thinkers (such as Albert of Saxony, John Buridan, Christophorus Clavius, William of Ockham, Alessandro Piccolomini, Proclus, Marcus Fabius Quintilianus, Thomas Reid, Andreas Rüdiger, Girolamo Saccheri and Christian Wolff) and relatively recent secondary sources (philosophers and logicians R. Lanier Anderson, E. Jennifer Ashworth, Lewis White Beck, Jonathan Barnes, Alonzo Church, Michael Friedman, Jaakko Hintikka, Henry Mendell, Daniel D. Merrill, Gabriel Nuchelmans, Paul Thom, etc.).

It goes without saying that putting forward a definition of syllogism is a delicate question which raises profound philosophical and logical problems. It is all the more difficult in the present context since many of the authors to whom Mancosu and Mugnai refer in this book have not left extensive explanations of their conception of the syllogism. It should be noted, however, that the authors skillfully manage to avoid this problem most of the time, and rarely does the angle from which they approach their subject of interest require them to reconstruct the precise conception of the syllogism defended by one or other of the thinkers under consideration.

The interested reader should be warned, however, that although the authors do their best to introduce and explain concepts that may be unfamiliar to the reader (especially those coming more from the disciplines of mathematics than from those of philosophy or logic in its broadest sense), it is nevertheless assumed that the reader already has a fairly detailed knowledge of Aristotelian syllogisms and Euclid's style of proof in the *Elements*.

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