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★**An introduction to functional analysis.**

*Cambridge University Press, London, 2020. xv+403 pp.*  
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This book covers the material of a standard functional analysis course offered at the end of an undergraduate course or at the beginning of a postgraduate course in mathematics. Key results that are a prerequisite for more in-depth study of specialised or applied branches of mathematical analysis such as harmonic analysis, differential equations or the calculus of variations are presented in an accessible but comprehensive way.

A first noteworthy fact is that no prior knowledge of measure theory or Lebesgue integration is assumed or required. Indeed, Lebesgue spaces are introduced as completions of the space of continuous functions with respect to an appropriate norm, while the usual construction of the Lebesgue integral is described in an appendix. Secondly, Hilbert spaces are not treated as a special case of Banach spaces, whose particular characteristics are mentioned in passing. Instead, the theory of Hilbert spaces is developed on its own before pushing the abstraction a notch higher by then turning to Banach spaces.

Written in the style of lecture notes (i.e., in a simple and concise style, with chapters composed of short sections, and with no attempt to enrich the exposition with comments on the history of the development of the ideas mentioned), this book represents a turnkey solution for anyone who has to teach a two-semester course in functional analysis. That said, it is also suitable, with some judicious pruning, for a one-semester course on Banach spaces.

The first two parts of the book, which cover around one hundred pages, serve to lay the foundations on which the notions of functional analysis which form the heart of this book are then built. Initially, since one of the main aims of this book is to establish infinite-dimensional analogues of familiar results for linear operators between finite-dimensional vector spaces, the author begins by briefly discussing some basic elements of abstract linear algebra. Starting with a reminder of what a vector space is, he goes on to show, using Zorn's lemma, that every vector space has a basis. He then digresses slightly to recall some important notions from the theory of metric spaces, such as the definitions of convergence, continuity, compactness and separability. Finally, the theory of normed vector spaces is explored in more detail, with a number of detailed examples. In this section, the book proceeds at a slower pace, delving into a number of highly important ideas such as those at the heart of Banach's fixed point theorem, Riesz's lemma, Weierstrass's approximation theorem, the Stone-Weierstrass theorem and the Arzelà-Ascoli theorem.

Part III develops the theory of Hilbert spaces in about a hundred pages. The author deals in turn with inner products, the Cauchy-Schwarz inequality, induced norms, orthonormal bases, orthogonal projections, the Riesz-Fréchet representation theorem and rudiments of spectral theory. The culminating point of Part III is the Hilbert-Schmidt (or eigenfunction expansion) theorem, which describes how a compact self-adjoint operator defined on a Hilbert space decomposes in terms of its eigenvalues and eigenfunctions. This major theorem is then applied, in the final chapter of Part III, to Sturm-Liouville eigenvalue problems.

The fourth and final part, also about a hundred pages long, presents the foundations of the theory of Banach spaces. It covers foundational results such as the Hahn-Banach theorem, the Kreĭn-Milman theorem, Baire's category theorem and its classical applications, Mazur's theorem, Helly's theorem that any bounded sequence in the dual of a separable Banach space has a weakly-\* convergent subsequence, and the result that

the closed unit ball in a reflexive Banach space is weakly sequentially compact. Three brief appendices totalling around thirty pages present the proofs of a handful of results, such as the Banach-Alaoglu theorem, which are useful but outside the scope of the main part of the book.

There are more than 200 exercises of varying degrees of difficulty in the 27 chapters that make up the book. In addition, complete solutions are presented at the very end of the book. Most of these exercises are fairly standard. A few, of a more uncommon nature, have been borrowed from other sources, in which case due credits are given.

*Frédéric Morneau-Guérin*