Martin Gardner et la question du réalisme

Réunion d'éducation en ligne 2023 de la SMC

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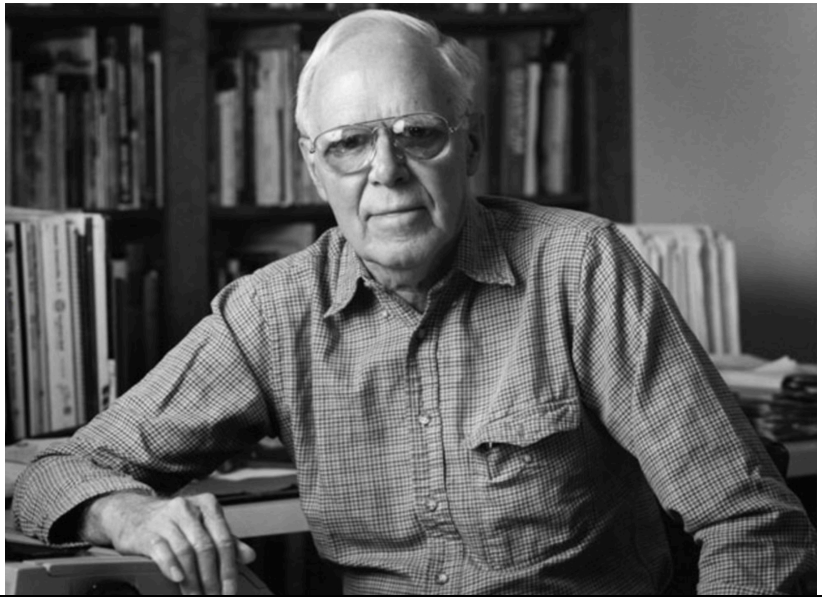
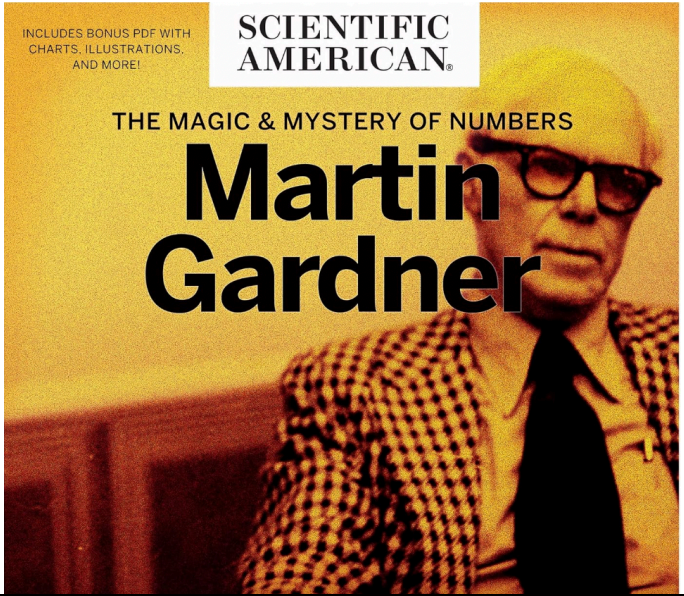
26 novembre 2023

Département Éducation, Université TÉLUQ

UNIVERSITÉ TÉLUQ

Qui est Martin Gardner ?

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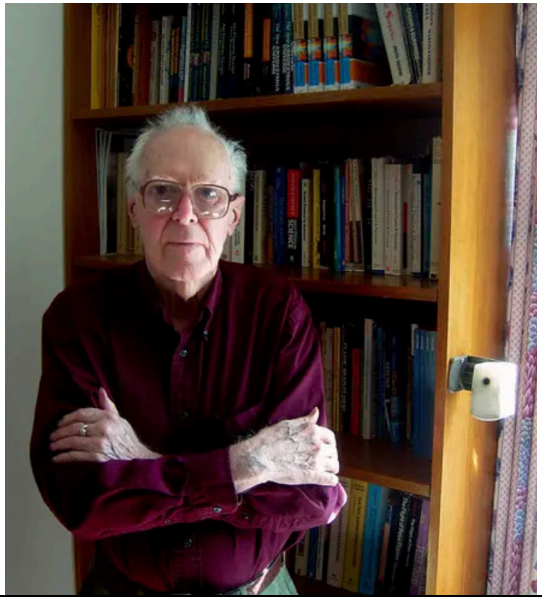


Testimonial 87: Richard Guy

"My comment (from *Winning Ways*): Martin Gardner ... who has brought more mathematics to more millions than anyone else. *for the Million*.)"

— Richard Guy, mathematician, Calgary, Canada (29 Jul 2015)

Testimonial 42: Dan Jarratt

Martin Gardner taught me about relativity when I was in grade school. The book occupies a proud place on my shelf even now.

— Daniel C. Jarratt, Ph.D. student in computer science at the University of Minnesota, MN, and vice president of research at PossibilityU (14 Feb 2014)

Testimonial 64: Jerry Grossman

"One of the influences on my decision to become a mathematician surely had to be Martin Gardner's first book of mathematical diversions, which my best friend gave me as a Bar Mitzvah present in 1961. I did a science fair project on hexaflexagons for the junior high school science fair, and that led to doing more advanced science fair projects on mathematical topics, which led to some prizes, which led to summer internships, which helped me get into a prestigious college, followed by a PhD from a prestigious graduate school and a 40-year (so far) career as an academic mathematician and author. Thank you!"

— Jerrold Grossman, mathematician, Rochester, MI, USA (9 Aug 2014)

Testimonial 68: Nick Berry

"I still remember, to this day, the location where, as a teenager, I purchased my first Martin Gardner books; it's one of those moments in life. There was life before discovery of Gardner, and there is the life after. Three books rapidly turned into half a dozen, then continued to grow from there. I still have them all proudly on my bookshelf. Like so many others who are reading this, when you consume his work, it's like he is writing just for you. His fascination for the subjects, his enthusiasm, enjoyment and interest shine through. It was wonderful to find someone that had such a passionate interest in things that I found so interesting."

— Nick Berry, Data Scientist at Facebook, Seattle, WA (5 Oct 2014)

Testimonial 70: Gordon Hamilton

"Martin Gardner was of course the primary reason to pick up *Scientific American*s. You can imagine my incredulity at discovering a twined-up pile of them back to the 1960s in a garage sale. I hefted them up to owner willing to open my teenage piggy bank, but the dusty, crumpled things were free! Joy! "

— Gordon Hamilton, mathematician, educator, board game designer, Canada (14 Oct 2014)

Testimonial 93: Hannah Fry

'I first came across Martin Gardner when I was 14 and found a book of his puzzles while on holiday at my aunt's house. From that moment on, I was hooked. It was beginning of many years spent gobbling up as much of his work as I could lay my hands on."

'Gardner had an extraordinary knack of igniting your imagination, of teasing you through twists and turns of his puzzles and of making mathematics come alive. I still have a shelf full of his books and will continue to treasure them for many years to come."

— Hannah Fry, mathematician and broadcaster, London, UK, (30 Jul 2015)

Testimonial 98: Andrew Barton

"I came across Martin Gardner's *Scientific American* column as a teenager, and after I'd been reading for a while I went to the library and systematically read through all his back columns. He helped inspire me to study mathematics, leading to a career based on logic and gaming. His *Annotated Alice* gave me new insight into the works of another favourite writer. "

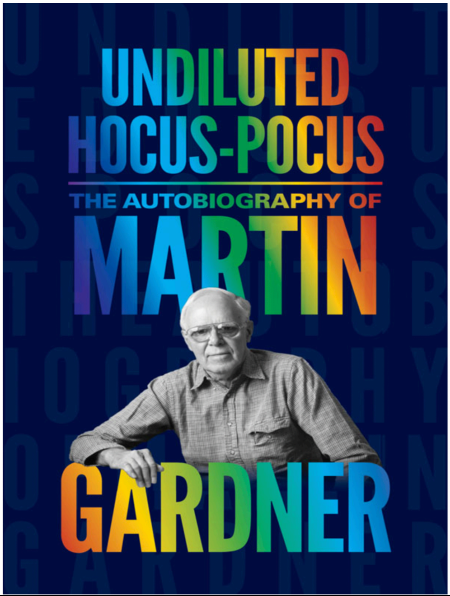
— Andrew Barton, logician and gamer, UK (11 Oct 2015)

Où loge Martin Gardner sur le plan philosophique ?

Philosophy of mathematics [edit]

Gardner wrote on the philosophy of mathematics.[142] He wrote negative reviews of *The Mathematical Experience* by Philip J. Davis and Reuben Hersh and *What Is Mathematics, Really?* by Hersh, both of which were critical of aspects of mathematical Platonism, and the first of which was well received by the mathematical community. While Gardner was often perceived as a hard-core Platonist, his reviews demonstrated some formalist tendencies.[143] Gardner maintained that his views are widespread among mathematicians, but Hersh has countered that in his experience as a professional mathematician and speaker, this is not the case.[144]

https://en.wikipedia.org/wiki/Martin_Gardner


UNDILUTED
HOCUS-POCUS
THE AUTOBIOGRAPHY OF
MARTIN
GARDNER

Roger and I share many opinions. We are both unashamed Platonists who believe mathematical theorems and objects are discovered, not created, with a reality independent of human cultures. We also agree that no computer of the kind we know

Revenir p. 180

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Undiluted Hocus-pocus (2013), ch. 15.


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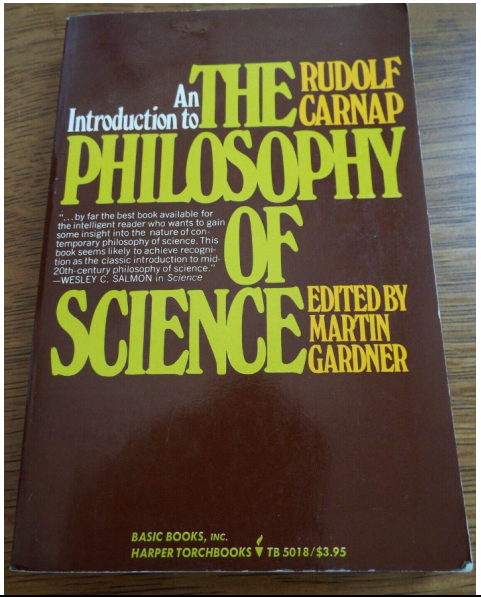
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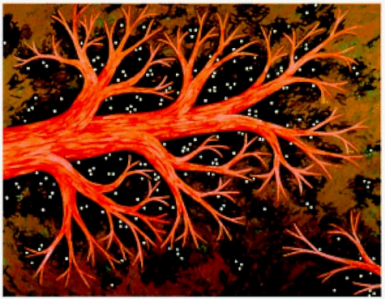
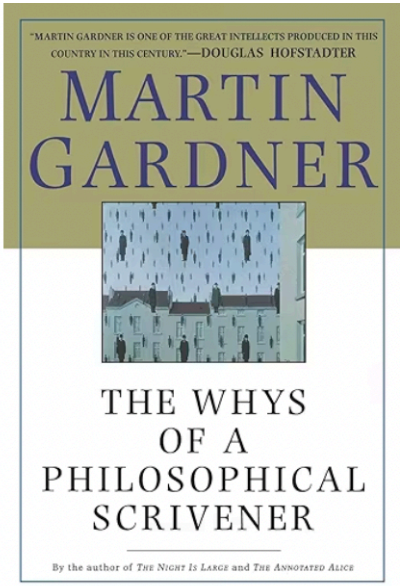
—Michael Dirda, *The Washington Post Book World*

MARTIN GARDNER

COLLECTED ESSAYS, 1938–1995



The Night Is Large

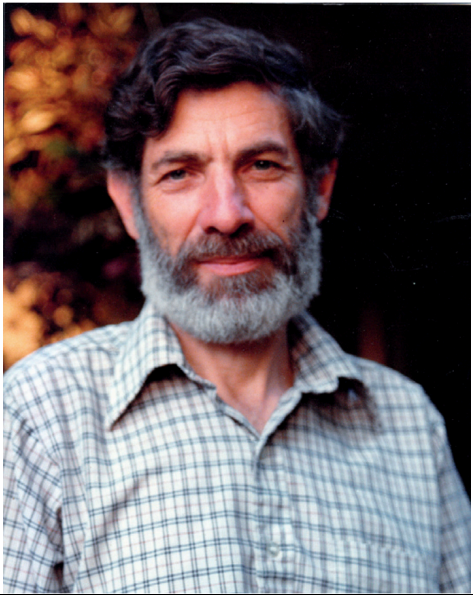

"MARTIN GARDNER IS ONE OF THE GREAT INTELLECTS PRODUCED IN THIS COUNTRY IN THIS CENTURY."—DOUGLAS HOFSTADTER
MARTIN GARDNER
THE WHYS OF A PHILOSOPHICAL SCRIVENER
By the author of THE NIGHT IS LARGE and THE ANNOTATED ALICE

The New York Review

Is Mathematics for Real?

Martin Gardner

August 13, 1981 issue



"Full-blooded Platonism" requires two distinct realms or universes – the physical, material world that includes human flesh and blood, and a separate, pre-existing, changeless universe of Ideas or Ideals or Abstractions. This myth does fit one aspect of the mathematician's daily experience. Yet it contradicts the standard world view, accepted by most scientists including mathematicians: the world is a single, united entity, with mind existing as a complex manifestation of matter in motion.

Platonism in the sense of existence of ideal entities, prior to human consciousness and matter, is tenable within a religious world-view— belief in a divine Mind. Martin Gardner the great journalist of mathematics, was such a believer.

Hersh, R. (2013). *Experiencing mathematics: What do we do, when we do mathematics?* (Vol. 83). American Mathematical Soc.. p.24-25.

Le platonisme mystique d’inspiration pythagoricienne

Platonisme mystique d'inspiration pythagoricienne

1. Clauses ontologiques :
   - 1.1 Les choses sensibles, sur lesquelles nous basons intuitivement notre jugement et que nous tenons pour réelles ne sont que des manifestations transitoires et imparfaites de la nature véritables des choses;
   - 1.2 Les concepts, notions, ou idées abstraites existent réellement dans un univers purement abstrait, sont immuables et universelles et forment les modèles (archétypes) des choses et formes que nous percevons avec nos organes sensoriels;
2. Clause épistémologique :
   - 2.1 La véritable connaissance ne peut être acquise que par l'intellect et la pensée rationnelle, plutôt qu'en s'appuyant uniquement sur la perception sensorielle.

Gardner adhère-t-il au platonisme mystique d'inspiration pythagoricienne ?

It is easy to caricature what mathematicians mean when they call themselves realists. They certainly do not suppose (I doubt if Plato did) that were we transported to some far-off realm we would see luminous objects floating about which we would recognize as pi, the square root of minus one, transfinite sets, pure circles, and so on; not symbols or models, but the undefiled universals themselves. Realists mean something less exotic. They mean that if all intelligent minds in the universe disappeared, the universe would still have a mathematical structure, and that in some sense even the theorems of pure mathematics would continue to be "true."

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Brian Davies, in his paper "Let Platonism Die" defines mathematical Platonism as the belief that mathematical entities exist 'in the mathematical realm outside the confines of space and time.' This is not what I or, I think, most mathematical Platonists believe. Aristotle, a mathematical realist, grabbed Plato's universals (redness, cowness, two-ness and so on) from a transcendental realm, and attached them to objects in space and time. The geometrical shape of a vase, for example, is 'out there', on the vase, not something floating outside Plato's cave.

Gardner, M. (2009). Is Reuben Hersh 'Out there'?. *Newsletter of the European Mathematical Society*, 23-24.

Le réalisme métaphysique (platonisme mathématique)

Réalisme métaphysique

1. Clauses ontologiques :
   - 1.1 Clause d'existence : Les entités physiques spatiotemporelles constitutives du monde extérieur, avec les propriétés qu'elles possèdent et les relations qu'elles entretiennent entre elles, existent réellement;
   - 1.2 Clause d'indépendance : Lesdites entités ne dépendent en aucune manière, pour leur existence ou pour leur nature, de la perception, de la pensée ou du langage;
2. Clauses épistémologiques :
   - 2.1 Clause de vérité-correspondance : La vérité est une sorte de relation de correspondance *unique* entre les termes du langage et les parties de la réalité;
   - 2.2 Clause de non-épistémicité : La vérité *transcende* radicalement notre pouvoir de la connaitre.

Gardner adhère-t-il vraiment au réalisme métaphysique ?

Donald E. Simanek: *One website refers to you as a "hard-core Platonist". What's your philosophical position on the quesiton of "Reality"?*
Martin Gardner: *Well, in the first place I'm a realist. I believe the external world exists independent of the human mind. I also believe that about mathematics. I think that mathematics exists independent of human intelligence. That's what makes me a Platonist. Actually I prefer the term "mathematical realist".*

Conversations with Martin Gardner, mai 2007.

Reuben Hersh, my old adversary, in a paper 'On Platonism' says this: My view of Platonism – always referring to the common, every-day Platonism of the typical working mathematician – is that it expresses a correct recognition that there are mathematical facts and entities, that these are not subject to the will or whim of the individual mathematician but are forced on him as objective facts and entities which he must learn about and whose independent existence and qualities he seeks to recognize and discover. Welcome, Professor Hersh, to the Plato club! All Platonists would agree completely with your remark.

Gardner, M. (2009). Is Reuben Hersh 'Out there'?. *Newsletter of the European Mathematical Society*, 23-24.

Thèse : Gardner adhère en bonne partie au réalisme putnamien

Réalisme interne (putnamien)

1. Clauses ontologiques :
   - 1.1 Clause d'existence : Les entités physiques spatiotemporelles constitutives du monde extérieur, avec les propriétés qu'elles possèdent et les relations qu'elles entretiennent entre elles, existent réellement;
   - 1.2 Clause d'indépendance : Lesdites entités ne dépendent en aucune manière, pour leur existence ou pour leur nature, de la perception, de la pensée ou du langage;
2. Clauses épistémologiques :
   - 2.1 La question *De quels objets le monde est-il fait ?* n'a de sens que dans une théorie ou une description;
   - 2.2 La *vérité* est une sorte de cohérence idéale de nos croyances entre elles et avec nos expériences telles qu'elles sont représentées dans notre système de croyances et non une correspondance avec des *états de choses* indépendants de l'esprit ou du discours;

Quelques principes qui sous-tendent le réalisme interne ou qui en découlent

1. Point de vue de Quine sur le savoir : notre savoir qui se déploie de manière holistique de telle sorte qu'il est toujours sous-déterminé par l'expérience;
2. L'expérience est toujours imprégnée de théorie : le schème conceptuel à partir duquel nous abordons l'expérience a des répercussions sur l'interprétation que nous faisons des données de nos sens;
3. Argument de l'absence de miracle : le fait que nos théories scientifiques soient des représentations approximativement adéquates de la réalité constitue la meilleure explication de leur succès instrumental;
4. Externalisme sémantique : le monde n'est pas divisé en espèces préalablement aux conventions linguistiques, c'est nous qui découpons la réalité; mais la signification d'un concept scientifique est principalement déterminée par son référent dans le monde réel.

Corollaires du réalisme interne en philosophie des mathématiques

1. Objectivité des énoncés mathématiques : étant donné que ceux-ci font partie intégrante de l'échafaudage de notre savoir, ils contribuent à l'élaboration de notre représentation globale de la réalité au même titre que les énoncés empiriques;
2. Argument d'indispensabilité des mathématiques pour la physique : nous n'avons pas moins de raisons de reconnaitre la vérité (approximative) des mathématiques auxquelles nous avons recours dans la formulation de nos théories physiques que de reconnaitre la vérité (approximative) de leur contenu empirique;
3. Approche quasi empiriste : une interaction de postulats et d'expérimentation quasi empirique nous mène à la formation de paradigmes contextuellement a priori;
4. Analyticité relative à un schème conceptuel : les énoncés fondamentaux des mathématiques sont si centraux qu'ils sont employés comme auxiliaires pour établir des prédictions dans nombre d'expériences sans être eux-mêmes mis en question par les résultats de ces expériences.

Notre savoir qui se déploie de manière holistique (holisme de signifiation)

> *Only in mathematics (and formal logic) are proofs absolutely certain. To say that two plus two equals four is like saying there are 12 eggs in a dozen. Changing four to any other integer would introduce a contradiction that would collapse the formal system of arithmetic.*

Gardner, M. (1997, October 12). What is Mathematics, Really?. *Los Angeles Times*

Notre savoir qui se déploie de manière holistique (holisme épistémologique)

> *No mathematician hesitates to speak of "existence proofs" about objects even when they are nowhere modeled, or known to be modeled, by the external world. And most mathematicians, including the very greatest, think of such objects as independent of the human mind, though not of course existing in the same way Mars exists.*

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Notre savoir est toujours sous-déterminé par l'expérience

> *Consider* $2^{1398269} - 1$. *Not until 1996 was this giant integer of 420, 921 digits proved to be prime [...]. A realist does not hesitate to say that this number was prime before humans were around to call it prime, and it will continue to be prime if human culture vanishes. It would be found prime by any extraterrestrial culture with sufficiently powerful computers.*

Gardner, M. (1997, October 12). What is Mathematics, Really?. *Los Angeles Times*

Notre savoir est toujours sous-déterminé par l'expérience

> *Does he really think that manipulating pebbles to prove, say, that 17 ia a prime is not a process going on out there, unconditioned by a given culture? Of course manipulating pebbles is culturally conditioned in the trivial sense that everything humans do are so conditioned. But that is not the deeper question. The primality of 17, in an obvious way, is out there in the behaviour of pebbles in much the same way that the elliptical orbit of Mars is out there, or the spiral of our galaxy. Hersh is so addicted to squeezing math inside the folkways that in his book What is mathematics, Really? he writes, so help me, that 8 plus 5 is not necessarily 13 because some skyscrapers have no floor 13. So if you go up 8 floors in an elevator, then go 5 more floors, you find yourself on floor 14. Is Hersh suggesting that in the subculture of some skyscrapers the laws of arithmetic are constantly violated?*

Gardner, M. (2009, June). Is Reuben Hersh 'Out there'. *Newsletter of the European Mathematical Society*. 23-24.

L'expérience est toujours imprégnée de théorie

> *To a realist, mathematical structure is mind-independent in two senses. The universe is not shapeless, but patterned in ways that are described by mathematics. In addition, mathematicians investigate purely abstract structures, defined by formal systems, which may or may not have applications to the physical world.*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

L'expérience est toujours imprégnée de théorie

> *Within the formal system of Euclidean geometry, as made precise by the great German mathematician David Hilbert and others, the interior angles of a triangle add to 180 degrees. As Hersh reminds us, this was Spinoza's favorite example of an indubitable assertion. I was dumbfounded to come upon pages on which Hersh brands this theorem uncertain because in non-Euclidean geometries the angles of a triangle add to more orless than a straight angle 180 degrees. Non-Euclidean geometries have nothing to do with Euclidean geometry. They are entirely different formal systems.*

Gardner, M. (1997, October 12). What is Mathematics, Really?. *Los Angeles Times*

L'expérience est toujours imprégnée de théorie

> *One of the greatest lessons that can be learned from the history of science is one of humility. Science may indeed be steadily learning more about the structure of the world, but surely what is known is exceedingly small in relation to what is unknown. There is no scientific theory today, not even a law, that may not be modified or discarded tomorrow.*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

Argument de l'absence de miracle

> *[W]hy do mathematical theorems fit the universe so accurately that they have enormous explanatory and predictive power? The authors call attention to Eugene Wigner's well-known paper, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." For a nonrealist this effectiveness is indeed an awesome mystery. And if mathematical concepts have no locus outside human culture, how has nature managed to produce such a boundless profusion of beautiful models of mathematical objects: orbits that are conic-section curves, snowflakes, coastlines that model fractal curves, carbon molecules that are tetrahedral, and on and on?*

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Argument de l'absence de miracle

> *Of course no realist denies that culture has a strong influence on how science operates, and even on what mathematicians do, but this is to say something trivial. The significant point is that in spite of cultural trends, regardless of science's corrigibility, the plain fact is that science gets ever closer to an understanding of nature. Otherwise, how can one explain the incredible successes of modern technology, and the equally incredible power of science to explain and predict? The accuracies of science rest on correspondences between statements about nature and the actual structure of a universe not made by us. Such statements are, obviously, part of culture because everything humans say and do is part of culture; but if the assertions of science did not correspond with great accuracy to the laws of nature, such accuracies would have to be viewed as a monumental set of miraculous coincidences.*

Gardner, M. (1983). The Whys of a Philosophical Scrivener. *William Morrow.*

Argument de l'absence de miracle

> *Although it is largely unrecognized, mathematics is, of course, part of our culture, because everything humans do is part of human culture. In a trivial sense mathematical objects and theorems are creations of human minds. The more profound mystery is why, after being created or discovered (either term is legitimate), they so beautifully fit the outside world. Physicist Eugene Wigner, who won a Nobel Prize in 1963, and who died in 1995, at age ninety-two, wrote a famous essay titled "The Unreasonable Effectiveness of Mathematics." As a Platonist, I believe that this seemingly miraculous correspondence arises because mathematical structure is the basis of the universe. Since our minds are part of the universe, is it so surprising that mathematics neatly fit a universe not made by us?*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

Argument de l'absence de miracle

> *Why does mathematics, obviously the work of human minds, have such astonishing applications to the physical world, even in theories as remote from human experience as relativity and quantum mechanics? The simplest answer is that the world out there, the world not made by us, is not an undifferentiated fog. It contains supremely intricate and beautiful mathematical patterns from the structure of fields and their particles to the spiral shapes of galaxies. It takes enormous hubris to insist that these patterns have no mathematical properties until humans invent mathematics and apply it to the outside world.*

Gardner, M. (1997, October 12). What is Mathematics, Really?. *Los Angeles Times*

Externalisme sémantique

> *The term by which a society designates the kinship of uncle-nephew is, of course, purely cultural. But the relationship itself, the denotatum of the sign, is clearly an aspect of the external world which would exist whether anyone gave a term to it or not. Dinosaurs had uncles and nephews long before a species evolved capable of describing the relationship in language symbols. Similarly with colors. […] And what are the consequences of affirming an external order? It follows that mathematical concepts, like colors and family linkages, have at least one foothold in a reality independent of the human species. It is true, of course, that an abstract triangle does not exist in the same sense a cow does, but this does not mean that the concept of triangle is an arbitrary cultural feature unconnected with the structure of the world.*

Gardner, M. (1950). Mathematics and the Folkways. *The Journal of Philosophy, 47*(7), 177-186.

Externalisme sémantique

> *For a mathematical realist a tree not only exists when nobody looks at it, but its branches have a "tree" pattern even when no graph theorist looks at them. Not only that, but when two dinosaurs met two dinosaurs there were four dinosaurs. In this prehistoric tableau "2 + 2= 4" was accurately modeled by the beasts, even though they were too stupid to know it and even though no humans were there to observe it. The symbols for this equality are, obviously, human creations, and our mental concepts of two, four, plus, and equals are by definition mind-dependent. If mathematical structure is taken to mean only what is inside the brains of those who do mathematics, it is as trivial to say all mathematics is mind-dependent as it is to define sound as a mental phenomenon, then proclaim that the falling tree makes no sound when nobody hears it.*

Gardner, M. (1981). Is mathematics for Real?. *The New York Review of Books, 13.*

Externalisme sémantique

> *Trees are independent of the minds of woodchoppers. Woodchopping theory is cultural, but the theory rests on something that is not.*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

Externalisme sémantique

> *There are two reasons for supposing that mathematical theorems are more than cultural conventions. One is that, whenever two cultures independently develop the same formal system, such as Euclidean geometry, they discover the same theorems. The second reason is that mathematics applies with eerie exactitude to the physical world. Any two cultures, isolated from each other, that develop a system for measuring the two sides of a right triangle and calculating the hypotenuse will discover the same Pythagorean rule, because that is how the world is structured. Of course, if they write down the rule or talk about it, their writing and talking will be mind-dependent in the same way that writing and talking about the moon is mind-dependent. But the moon itself, and the fact that it is spherical, is not mind-dependent.*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

Externalisme sémantique

> *Purely abstract structures such as circles, triangles, groups, numbers, infinite sets, higher dimensions, and thousands of other mathematical objects seems to have a peculiar existence of their own, independent of both the outside world and the human mind – even though they don't "exist" in the way stones and stars exist.* *Like almost all mathematicians, I am a Platonist who believes that mathematical objects and theorems are as much discovered as scientific laws. I believe that a huge number with, say, a million digits is either a prime of a composite before any human proves it one way or the other.*

Gardner, M. (1997). *The night is large: collected essays*, 1938-1995. Macmillan.

Externalisme sémantique

> *Indeed, the universe is saturated with models of nearly all of mathematics. Even a topologist can prove that bisecting a Klein bottle produces two Moebius bands of opposite handedness by making a crude model out of an envelope, then cutting it in half. Complex numbers and derivatives may not have material models, but they also are embedded throughout the universe. […] The Mandelbrot set is not outside space and time. It exists on computer screens. Does an anti-realist believe that a mathematician exploring properties of the Mandelbrot set is really exploring a structure inside his brain because his eyes and brain are seeing the screen, or that he is exploring part of his culture because his culture built the computer? Such statements are the same kind of language distortion as claiming that astronomers are not studying patterns, 'out there' because telescopes are part of culture, not to mention all of astronomy as well.*

Gardner, M. (2009, June). Is Reuben Hersh 'Out there'. *Newsletter of the European Mathematical Society.* 23-24.

Argument d'indispensabilité des mathématiques pour la physique

> *A moment's reflection and one will realize that any imaginable society, to exist at all, must constantly make use of elementary mathematical laws which are everywhere the same, but so commonplace we are seldom aware of them. For example, to exist it is necessary to eat, and to eat it is necessary that something be placed in the mouth. But the fact that a smaller object can be pushed through a larger orifice is a fact of geometry. A tribe which acted on the reverse assumption, i.e., that only larger objects could be placed through smaller holes, would be acting on a law which, however well grounded it might be in the mythology of the culture, would be so poorly grounded in mathematical reality that the tribe would quickly die of starvation.*

Gardner, M. (1950). Mathematics and the Folkways. *The Journal of Philosophy, 47*(7), 177-186.

Argument d'indispensabilité des mathématiques pour la physique

> *In the light of today's physics the entire universe has dissolved into pure mathematics. The cosmos is made of molecules, in turn made of atoms, in turn made of particles which in turn may be made of superstrings. On the pre-atomic level the basic particles and fields are not made of anything. They can be described only as pure mathematical structures. If a photon or quark or superstring isn't made of mathematics, pray tell me what it i s made of?.*

Gardner, M. (2001). Is mathematics "Out there"?. *The Mathematical Intelligencer, 23*(1), 7-8.

Approche quasi-empirique

> *It is true there is no a priori reason why a geometrical postulate, such as the famous parallel postulate, must be true. A coherent non-Euclidean system can be constructed which will violate this postulate, and the discoveries of modern physics suggest that such a system is a sounder basis on which to make calculations involving high speeds and astronomical distances. But on the level of ordinary speeds and distances, we live very much in an old-fashioned [...] Euclidean world which does not vary in any measurable respect from one geographical area to another. If a tribesman wishes to shoot a bird with a bow and arrow, it is necessary for him [...] to aim the arrow at the bird in strict accord with Euclidean and Newtonian laws and not, say, to shoot the arrow in the opposite direction on the grounds that straight lines, prolonged indefinitely in a closed 4^{th}-dimensional continuum, will circle the cosmos and return to the starting point.*

Gardner, M. (1950). Mathematics and the Folkways. *The Journal of Philosophy*, *47*(7), 177-186.

Analyticité relative à un schème conceptuel

> *To blur the distinction between analytic and synthetic truth (as Willard Van Orman Quine and others have done) is to blur the difference between science and mathematics.*

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Analyticité relative à un schème conceptuel

> *In an article in Eureka (March 1988), [Hersh] discusses several "myths" which he claims have been mistakenly defended by great mathematicians. [...] Myth 4: "Mathematics possesses a method called 'proof' ... by which one attains absolute certainty of conclusions, given the truth of the premises." Can Hersh be serious when he calls this a myth? In mathematics, unlike in science, proof is the essence. Given the symbols, and the formation and transformation rules of a formal system, all theorems are tautologies. They are, as Kant was the first to say, analytic, not synthetic.*

Gardner, M. (2001). Is mathematics "Out there"?. *The Mathematical Intelligencer, 23*(1), 7-8.

Analyticité relative à un schème conceptuel

> *Now a large portion of mathematics is analytic, and where it is, there is no harm in speaking of certainty. The truth of 2 + 2 = 4 does not depend (as John Stuart Mill contended) on the pleasant fact that two fingers plus two fingers make four fingers. It follows from the way terms are defined in a formal system that constructs integers.*

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Analyticité relative à un schème conceptuel

> *Moreover, thanks to the work of Kurt Gödel (whose Platonic realism was extreme) we know that in any formal system complicated enough to include arithmetic there are theorems that cannot be proved within the system. The structure of a brick may indeed have mathematical properties that can never be completely captured within a deductive system. None of this touches the realist view that the brick and its properties are independent of human minds, and that where proofs are simple enough to be formalized they can be considered "certain" in a way that does not apply to any scientific claim.*

Gardner, M. (1981). Is Mathematics for Real?. *The New York Review of Books, 13.*

Analyticité relative à un schème conceptuel

> *Only in pure logic and mathematics can statements be deemed absolutely certain, but for this kind of truth a stupendous price is paid. The price is that such statements say nothing about the world.*

Gardner, M. (1983). The Whys of a Philosophical Scrivener. *William Morrow.*

Merci !

Des questions?