

# Resonator with variable-reflectivity output coupler

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A novel resonator is proposed, in which the polarizing properties of standard grating and polarizing components (such as Brewster angle windows) inside the cavity are used to vary continuously the output coupling over a wide range of values. An analysis of the polarization properties of the modes of such a resonator is presented, and practical suggestions are made about the choice of the polarizing components. The results of an experiment involving a TEA CO<sub>2</sub> laser are shown to verify some of the conclusions of the analysis.

## I. Introduction

Line selective cavities are usually obtained by replacing the mirror at one end of a resonator by a reflection grating (most likely blazed in a way to maximize first-order reflectivity at the desired wavelength); a partial reflector at the other end provides the (hopefully) optimal output coupling.

An interesting alternative to that scheme is using zero-order (or an order higher than first) reflection on the grating for output coupling along with a totally reflecting mirror at the other end of the cavity.<sup>1,2</sup> Among the advantages of this configuration is the reduction of the losses, as the zero-order reflection is always present even with gratings specifically designed for maximum efficiency at first order. (For gratings used with CO<sub>2</sub> lasers, it is typically of the order of a few percent.) An interesting application was recently presented<sup>3</sup>: in a collinear optical pumping configuration, a grating acted simultaneously as an output coupler for the pump laser (a cw CO<sub>2</sub> laser) and a mirror for the pumped laser (a 12- $\mu$ m NH<sub>3</sub> laser), so that part of the NH<sub>3</sub> cavity coincided with the CO<sub>2</sub> cavity, greatly reducing the number of optical components required. In that experiment the pump laser was operated on a low-gain line; hence a standard grating could provide the optimal reflectivity when used in Littrow configuration. Adapting that scheme to higher-gain lines of

the cw CO<sub>2</sub> laser, e.g., 9R(30) or 9R(16), which are extensively used for NH<sub>3</sub> laser pumping, or to any of the TEA CO<sub>2</sub> laser lines would require specifically blazed gratings, as the optimum values for the reflectivity lie in the 70–95% range for cw lasers and 30–50% for TEA lasers.

The purpose of this paper is to show that it is possible to obtain a variable-reflectivity output coupler from a commercially available grating in a very simple way, so that the optimum coupling for any particular cavity can easily be attained. We first show that by simply rotating polarizing components, which are in most cases already present in a laser cavity (e.g., Brewster-angle windows), it is possible to select a wide range of zero-order reflectivities. The choice of the best-suited polarizing components is then discussed along with some important properties of the laser field polarization. Finally, the results of an experiment with a TEA CO<sub>2</sub> laser operated with this resonator are presented, and further applications are suggested.

## II. Polarizing Properties of Gratings

Polarizing properties of gratings are well known, but in practice these components are normally positioned in a way to minimize zero-order reflection, i.e., with polarization perpendicular to the grating lines, so that one is usually concerned only by the value of the first-order reflectivity for that polarization. One can, however, wonder about the case of a plane wave incident on a grating with its polarization making an arbitrary angle with the grating lines. By considering separately the two components (perpendicular and parallel to the lines) of the field polarization, it is easy to show that if the reflection coefficients (in amplitude) for the two polarizations are  $r_i$  and  $r'_i$ , respectively ( $i = 0$  or  $1$ , the order of reflection), the expression for these coefficients (in intensity) for an arbitrary angle  $\alpha$  between the polarization and the grating lines is

$$R_i(\alpha) = r_i^2 \sin^2 \alpha + r'_i{}^2 \cos^2 \alpha. \quad (1)$$

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A typical example of these relations is shown in Fig. 1, where these coefficients were measured for a particular wavelength [the 9R(30) line of a CO<sub>2</sub> laser] and a standard grating at different angles  $\alpha$  and where the values of  $r_i$  and  $r'_i$  were determined by a best fit.

### III. Resonator with Rotated Polarizing Components

Now we try to answer the following question: what will be the overall losses of a resonator in which a polarizing component is positioned in a way that its plane of maximum transmission makes an angle  $\alpha$  with the grating lines (Fig. 2). The obvious (and wrong) answer is to calculate  $R_1(\alpha)$  with Eq. (1) and to use the value obtained as the reflectivity of the grating. Doing so, one has forgotten to take into account the change of the plane of polarization occurring on reflection on the grating, so that if one assumes that the polarization is initially parallel to the plane determined by the polarizing component in a manner to minimize the losses, these losses can become much higher when the field goes back through the same component.

One thus has to answer the two following questions: what is the orientation of the field polarization that minimizes the losses on a round trip? Does the polarization remain the same after this round trip, and, if not, for what angle does it remain unchanged?

The problem can be formulated in the following way: let  $\hat{A}$  be the Jones vector describing the polarization of the field immediately before the grating:

$$\hat{A} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}, \quad (2)$$

where  $\theta$  is the angle between polarization and the  $x$  axis.

We choose the  $x$  axis as the axis of maximum transmission of the polarizing component; let this transmission be equal to unity, as in the case of Brewster-angle windows, and  $\Gamma$  the transmission (in amplitude) along the  $y$  axis. For a resonator with  $N$  Brewster-angle windows, assumed to be all parallel, with amplitude transmission coefficients  $\gamma_i$  ( $i = 1$  to  $N$ ), the value of  $\Gamma$  for a round trip is

$$\Gamma = \prod_{i=1}^N \gamma_i^2, \quad \text{with } \gamma_i = (2n_i/n_i^2 + 1)^2, \quad (3)$$

where  $n_i$  is the refractive index of the  $i$ th window.

For example, with two NaCl ( $n = 1.5$ ) windows, one finds  $\Gamma = 0.53$ , and  $\Gamma = 0.0338$  for two NaCl and two ZnSe ( $n = 2.4$ ) windows.

After a round trip, the polarization vector  $\hat{A}$  transforms into

$$\hat{A}' = M\hat{A}, \quad (4)$$

where

$$\hat{A}' = \lambda \begin{pmatrix} \cos\theta' \\ \sin\theta' \end{pmatrix} \quad (5)$$

and

$$M = \begin{bmatrix} r_1 \sin^2\alpha + r'_1 \cos^2\alpha & (r_1 - r'_1) \sin\alpha \cos\alpha \\ \Gamma(r_1 - r'_1) \sin\alpha \cos\alpha & \Gamma(r_1 \cos^2\alpha + r'_1 \sin^2\alpha) \end{bmatrix} \quad (6)$$

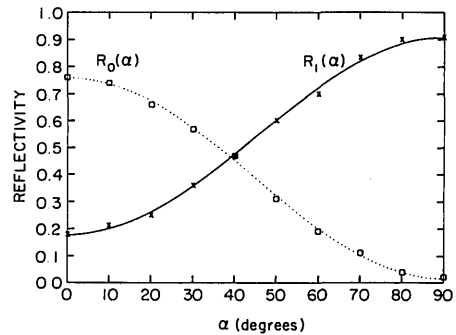


Fig. 1. Measured reflectivity (in intensity) at order zero  $\square$  and first order  $\times$  and results of a best fit from Eq. (1): 150-line/mm (PTR ML-303) grating, R(30) line of CO<sub>2</sub> laser. Values obtained:  $r_0 = 0.130$ ;  $r'_0 = 0.871$ ;  $r_1 = 0.953$ ;  $r'_1 = 0.418$ .

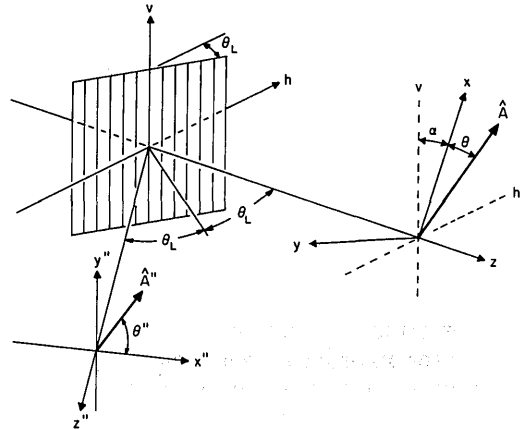


Fig. 2. Geometry of resonator with rotated polarizing components. The  $x$  axis is the maximum transmission axis of these components.

is the Jones matrix for a round trip; let us call  $\lambda$  the feedback parameter.

With this formulation, the eigenvector of matrix  $M$  corresponds to the polarization that reproduces itself after one round trip, and  $1 - \lambda^2$  gives the total losses (internal plus coupling). One should be careful to remember that the solution thus obtained represents the polarization in one part of the resonator only and that the orientation of the polarization changes all along the way on a round trip.

To answer the second question, one has to find the value of  $\theta$  that maximizes  $\lambda$  irrespective of the value of  $\theta'$ . Unfortunately, the relatively simple expressions involved so far lead to cumbersome equations when one tries to solve the extremum or eigenvalue problems mentioned above. Except in the case  $\Gamma \ll 1$ , where much simpler expressions arise, it is better to investigate numerically the behavior of the solutions. The most important conclusions of this analysis are the following:

Except for the limiting cases  $\alpha = 0^\circ$  and  $\alpha = 90^\circ$ , the angle  $\theta_{\min}$  of the polarization that minimizes the losses is not equal to  $\theta_{\text{eigen}}$ , the angle of the polarization that reproduces itself after one round trip (Fig. 3).

If the polarizing components are sufficiently more polarizing than the grating, i.e.,  $\Gamma < r'_1/2r_1$ , a field structure with its initial polarization minimizing the losses converges toward the eigenvector after a few round trips; the smaller the value of  $\Gamma$ , the faster the convergence. In this situation, one can reasonably assume that the eigenpolarization is the resulting polarization of the laser field.

As long as  $\Gamma < r'_1/r_1$ , it is possible to select any output coupling between  $r'_0$  and  $r'_2$ , the only setback being a slight increase of the internal losses for intermediate values (Fig. 4). These losses (apart from the natural losses of the grating due to diffusion and finite conductivity, which were  $\sim 7\%$  in our case) arise from the fact that the field eigenpolarization is not in general parallel to the maximum transmission axis of the polarizing components, and that in all cases it is rotated after reflection on the grating, so that the field suffers significant attenuation on passing back through the same components. One finds also that for smaller values of  $\Gamma$  these losses decrease, reaching a minimum (for a given value of  $\alpha$ ) at  $\Gamma = 0$ . This last property can be understood in terms of competition between two polarizing optical components, the more polarizing being able to pull the field polarization toward its maximum transmission axis. Figure 5 shows an example of the situation for two values of  $\Gamma$ , one lower and the other higher than the value of  $r'_1/r_1$ . In the first case, the polarization  $\theta_{\text{eigen}}$  remains always within  $20^\circ$  from the maximum transmission axis of the windows as these are rotated relative to the grating, while in the other it remains almost perpendicular to the grating lines.

As mentioned above, the case  $\Gamma \ll 1$  leads to somewhat simpler results. If one represents matrix  $M$  by

$$M = \begin{pmatrix} A & B \\ \Gamma B & \Gamma D \end{pmatrix}, \quad (7)$$

where  $A$ ,  $B$ , and  $D$  are given by Eq. (6), one obtains

$$\theta_{\text{eigen}} \approx \tan^{-1}(\Gamma B/A), \quad (8)$$

$$\theta_{\text{min}} \approx \tan^{-1}(B/A), \quad (9)$$

$$\lambda_{\text{eigen}} \approx A + \Gamma B^2/A, \quad (10)$$

$$\lambda_{\text{max}} \approx (A^2 + B^2)^{1/2}. \quad (11)$$

#### IV. Practical Considerations

By considering the results obtained with a large number of test values, it is possible to make some practical suggestions. First, the value of  $\Gamma$  should be a fraction of the ratio of the first-order reflection coefficients  $r'_1/r_1$ . If this condition is not fulfilled, the resulting polarization inside the cavity, as stated before, follows the orientation of the grating, tending to remain perpendicular to its lines. To force the polarization to become parallel to the grating lines, enabling the first-order reflectivity to reach its lowest value  $r'_1$ ,  $\Gamma$  has to be less than  $r'_1/r_1$ . Again Fig. 5 illustrates this effect, showing clearly the necessity of the condition  $\Gamma < r'_1/r_1$ . It should be noticed that two NaCl windows were not sufficient in our case to obtain the full range of reflectivities, as the corresponding value of  $\Gamma$  is 0.53, still higher than  $r'_1/r_1$  (0.44 in our experiment).

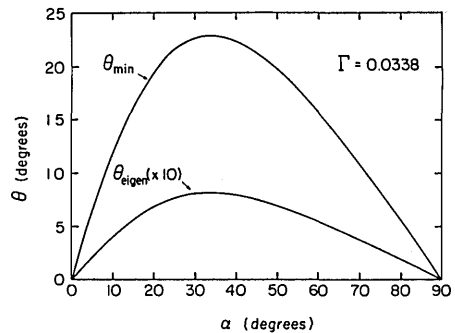


Fig. 3. Difference between eigenpolarization angle  $\theta_{\text{eigen}}$  and minimum losses angle  $\theta_{\text{min}}$ . Note change of scale for  $\theta_{\text{eigen}}$ .

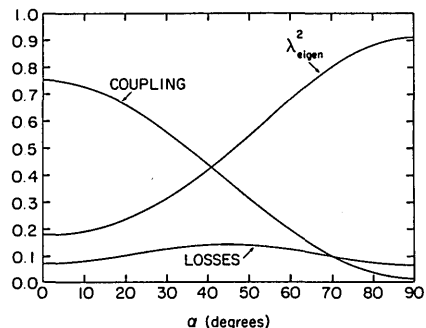


Fig. 4. Variation of internal losses, output coupling, and feedback parameter showing the wide range of coupling coefficients and the slight increase of the losses for intermediate values of  $\alpha$ .  $\Gamma$  has the same value as in Fig. 3.

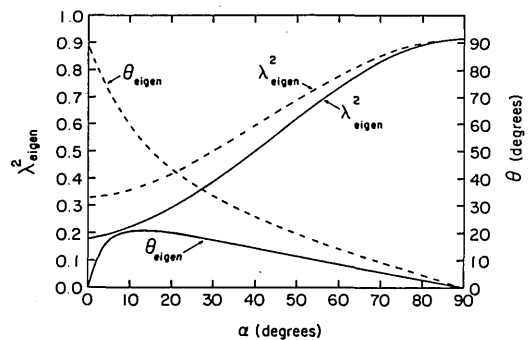


Fig. 5. Difference in behavior of polarization and feedback parameter for values of  $\Gamma$  smaller and larger than  $r'_1/r_1$  equal to 0.44 in this example. Values of  $\Gamma$  are 0.40 (—) and 0.60 (---), respectively. Note the dramatic change in the case of  $\theta_{\text{eigen}}$ .

Second, if it is important in a particular experiment to keep the losses at their lowest value, one should try to make  $\Gamma$  as small as possible. Figure 6 shows the value of the losses in the case of four NaCl windows for a value of  $\Gamma = 0.28$  compared with the case of a much smaller value of  $\Gamma$  and for the limiting case  $\Gamma = 0$ . One should then consider selecting high-refractive-index materials, such as ZnSe or Ge in the case of  $\text{CO}_2$  lasers.

Finally, one should not forget that even with a value of  $\Gamma$  small enough to cause the polarization to be almost identical to that existing in a regular resonator (i.e., parallel to the maximum transmission axis of the windows), the polarization outside the cavity is rotated

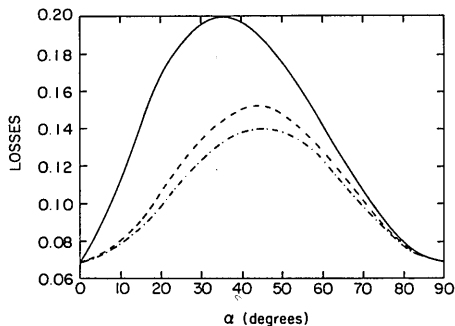


Fig. 6. Internal losses for three values of  $\Gamma$ : 0.28 (—); 0.06 (---); and the limiting value  $\Gamma = 0$  (- · -)

relative to that axis. One can easily calculate the new orientation from knowledge of the eigenvector  $\hat{A}$ . Let  $\theta''$  be the angle of the polarization outside the resonator, relative to the  $x''$  axis, which in this case is perpendicular to the grating lines (see Fig. 2); one finds

$$\tan \theta'' = \frac{r'_0 \cos(\alpha + \theta)}{r_0 \sin(\alpha + \theta)}, \quad (12)$$

where  $\theta$  is the orientation of eigenvector  $\hat{A}$ .

## V. Experimental Results

To verify our calculations, we performed an experiment using a TEA  $\text{CO}_2$  laser in hybrid configuration. The cavity thus contained four Brewster-angle windows, two of NaCl and two of ZnSe, giving a value of 0.0338 for  $\Gamma$ . The laser was operated on the  $9R(30)$  line, for which the values of the reflectivities of the grating had been measured. The optimum coupling, as determined by maximizing the energy output, was obtained with the windows rotated  $60^\circ$  relative to their

usual position, i.e., with  $\alpha = 30^\circ$ . This corresponded to a value of 0.31 for  $\lambda^2$ , slightly lower than in the case of an usual cavity. (The Ge output coupler with first-order reflection on the grating gives  $\lambda^2 = 0.33$ .) The polarization of the field outside the cavity was also measured, and the value obtained ( $\theta'' = 84 \pm 1^\circ$ ) matched closely the value obtained from Eq. (12).

## Conclusion

We have proposed a novel resonator featuring a variable-reflectivity output coupler that requires only standard commercially available optical components. An analysis of the properties of such a resonator was made, showing that a wide range of reflectivities can be attained if one uses well-chosen components inside the cavity. The questions of the losses and the resulting polarization (inside and outside the cavity) were also discussed. Finally, the results of an experiment verifying the above conclusions were presented.

Apart from its interest in collinear optical pumping schemes, this new resonator offers many possibilities. We are planning to use this technique to control to a high degree of accuracy the output power of a cw  $\text{CO}_2$  laser simply by inserting a feedback-controlled rotating Brewster-angle window inside the cavity for use in a differential absorption experiment.

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