

# Banach Algebras and Applications 2019

## Plenary Speakers

Speaker: **Michael Brannan** (Texas A&M University)

Title: *Unitarizable groups and quantum groups*

Abstract: A locally compact group  $G$  is called unitarizable if every uniformly bounded representation of  $G$  on a Hilbert space is similar to a unitary representation. In 1950, Day and Dixmier proved that amenable groups are always unitarizable, and it has been an open question since then to determine if the converse to this result holds. I.e., is every unitarizable group amenable? In this talk, I will give a brief overview on the status on the unitarizability=amenability problem for groups, and then I will shift my focus to the question of whether a quantum version of the Day-Dixmier unitarizability theorem holds: Is every amenable locally compact quantum group unitarizable? For some classes of quantum groups (e.g. group duals, amenable Kac algebras) current evidence suggests the answer should be yes. However, for certain well known classes of amenable quantum groups (including all Drinfeld-Jimbo-Woronowicz  $q$ -deformations of classical compact groups) we show that the answer is no. (This is joint work with Sang-Gyun Youn.)

Speaker: **Yemon Choi** (Lancaster University)

Title: *Approximate homomorphisms: some old and new results*

Abstract: The natural notion of an approximate homomorphism between two Banach algebras  $A$  and  $B$  is a bounded linear map  $\phi : A \rightarrow B$  for which the quantity

$$\text{def}(\phi) = \sup\{\|\phi(xy) - \phi(x)\phi(y)\| : x, y \in A, \|x\| \leq 1, \|y\| \leq 1\}$$

is “small”. We may think of this as measuring the worst-case scenario for “local” tests of the homomorphism property. The associated stability problem (in the sense of Ulam) is to find conditions on  $A$  and  $B$  which ensure that an approximate homomorphism is actually close in norm to a genuine homomorphism.

The study of such stability problems, for particular classes of Banach algebras, has a long history, with various researchers independently arriving at very similar questions and results. In this talk I will present a small selection of some of the known results, with particular emphasis on a pair of papers by B. E. Johnson from the 1980s that arguably deserve to be better known. I will then sketch some more recent results, obtained with various collaborators, in a few settings that may be of interest to different groups of specialists, such as: weighted semigroup algebras; Fourier algebras of locally compact groups; and algebras of operators on Banach spaces. The intention is to give a sample rather than to be exhaustive.

Speaker: **H. G. Dales** (Lancaster University)

Title: *Representations of Banach lattice algebras*

Abstract: I shall define a Banach lattice algebra and give several key examples, including the group algebra of a locally compact group. I shall then discuss and prove a representation theorem for these algebras.

This talk is based on joint work with Marcel de Jeu, Leiden.

Speaker: **Matthew Daws** (University of Central Lancashire)

Title: *Analytic generators of automorphism groups*

Abstract: We will discuss the (somewhat classical) notion of the “analytic generator” of a one-parameter group of isometries on a Banach space. The generator is an (in general) unbounded operator which is closed. Such objects are most often studied in relation to automorphism groups of operator algebras. In this case, the graph of the analytic generator is a non-selfadjoint algebra. In concrete situations, the resulting object is something like a Hardy space, and has links to Arveson’s theory of sub-diagonal algebras, while not being precisely these things. We are particularly interested in the case when we have an automorphism group of a von Neumann algebra which restricts to a sigma-weakly dense  $C^*$ -subalgebra; here the analytic generator of the  $C^*$ -algebra group is sigma-weakly dense in that for the von Neumann algebra. We prove a Kaplansky Density type result in this setting. This setup occurs in the theory of locally compact quantum groups, in particular, when the antipode is unbounded, which was our original motivation. Time allowing, we will make some comments in this direction.

Speaker: **J. Esterle** (Université de Bordeaux)

Title: *A holomorphic functional calculus for finite families of commuting semigroups*

Abstract: We consider here finite families of commutative families of strongly continuous semigroups of multipliers on a commutative Banach algebra  $\mathcal{A}$  which possesses dense principal ideal and is weakly cancellative, which means that  $a\mathcal{A} \neq \{0\}$  for every  $a \in \mathcal{A} \setminus \{0\}$ . The semigroups under consideration are either one-parameter semigroups defined on the half-line or semigroups holomorphic on open sectors.

This functional calculus takes value in the algebra  $\mathcal{QM}(\mathcal{A})$  of quasimultipliers on  $\mathcal{A}$ , introduced by the author at the Long-Beach conference on Banach algebras and applications in 1981. A renormalization process allows to embed the given Banach algebra  $\mathcal{A}$  into a Banach algebra  $\mathcal{B}$  which has the same quasimultiplier algebra as  $\mathcal{A}$ , but for which the given semigroups are bounded near the origin for semigroups defined on half-line and bounded near the origin on all sectors strictly smaller than their sector of definition for semigroups holomorphic on open sectors.

This allows to use the Cauchy formula to define a functional calculus on the generators of the semigroup with values in the multiplier algebra  $\mathcal{M}(\mathcal{B})$  for functions satisfying some  $H^1$  type conditions on a family of domains satisfying some suitable geometric conditions with respect to the “Arveson spectrum” of the generators of the semigroups under consideration. An extension to this functional calculus to the  $H^\infty$  spaces on the same class of domain takes values in the algebra  $\mathcal{QM}_r(\mathcal{B}) = \mathcal{QM}_r(\mathcal{A})$  of “regular” quasimultipliers on  $\mathcal{B}$  and  $\mathcal{A}$ . Finally this functional calculus is extended to a suitable version of the Smirnov class on the same domains. This extension takes value in the quasimultiplier algebra  $\mathcal{QM}(\mathcal{B}) = \mathcal{QM}(\mathcal{A})$ , and the generators of the given semigroups can be interpreted as the images of the coordinate function via this functional calculus.

The notions of Arveson ideal and the Fourier-Borel and Cauchy transforms play an important role in this holomorphic functional calculus.

Speaker: **Matthew Kennedy** (University of Waterloo)

Title: *Noncommutative Choquet theory*

Abstract: I will present a new framework for noncommutative convexity and noncommutative function theory, along with a corresponding noncommutative Choquet theory that generalizes much of classical Choquet theory. These ideas provide a new perspective on operator systems (including  $C^*$ -algebras) and completely positive maps. I will discuss several applications, including an integral representation theorem that generalizes Choquet's theorem. I will also introduce a notion of noncommutative Choquet simplex, which generalizes the classical notion of Choquet simplex and plays an analogous role in noncommutative dynamics.

This includes joint work with Ken Davidson and Eli Shamovich.

Speaker: **Anthony To-Ming Lau** (University of Alberta)

Title: *Fixed point set for semigroup of mappings on Banach spaces related to harmonic analysis*

Abstract: In this talk, I shall describe the set of fixed points for a semigroup of affine mapping acting on a closed convex subset of a Banach space with applications to harmonic analysis on groups or semigroups.

Speaker: **Niels Laustsen** (Lancaster University)

Title: *Closed ideals of the algebra of bounded operators on a Banach space*

Abstract: Significant progress has been made in our understanding of the lattice of closed ideals of the Banach algebra  $\mathcal{B}(X)$  of bounded operators on a Banach space  $X$  over the last decade. I shall survey some highlights of this development and then focus on the outcomes of an ongoing collaboration with Kevin Beanland (Washington and Lee University, VA, USA) and Tomasz Kania (Czech Academy of Sciences) in which we study the closed ideals of  $\mathcal{B}(X)$  in the case where  $X$  is either Tsirelson's Banach space or a Schreier space of finite order. No prior knowledge of these spaces will be assumed in the talk.

Speaker: **Hun Hee Lee** (Seoul National University)

Title: *Twisted Fourier space, multipliers and quantum information theory*

Abstract: In this talk we introduce a generalization of the Fourier algebra on a locally compact group, namely the twisted Fourier space. We will begin with a 2-cocycle on the group and continue with several twisted algebras including twisted group von Neumann algebra, whose canonical predual is the twisted Fourier space. Unlike the non-twisting case the new space does not carry a natural Banach algebra structure, but, at least, it is a bimodule of the usual Fourier algebra. However, the twisting allows us to include well-known quantum spaces such as non-commutative torus.

As the first project on this twisted Fourier space we will consider twisted version of a result of Bozejko/Losert/Ruan saying that  $M_{cb}A(G) = B(G)$  (or  $MA(G) = B(G)$ ) characterizes amenability of a locally compact group  $G$ . Note that the above result requires a thorough understanding of a somewhat mysterious proof of Losert.

Secondly, we will consider a possible application to quantum information theory, where we are interested in quantum states, i.e. a positive operators acting on Hilbert spaces with trace 1. Operators are, in general, difficult to handle, so that we prefer to find associated functions possibly keeping all the information intact. This is what quantum researchers have been doing when they investigated  $n$ -mode Bosonic quantum state under the name of characteristic functions and Wigner functions. In this talk we will see that this state  $\leftrightarrow$  characteristic function correspondence is nothing but a twisted Fourier transform on the group  $R^{2n}$ , where we may possibly apply abstract

harmonic analysis tools for understanding certain quantum states. We will focus on a few old/new observations where the above point of view plays an important role.

Speaker: **Matthias Neufang** (Carleton University and University of Lille)

Title: *Non-commutative Fejér theorems, and the projective tensor product of  $C^*$ -algebras*

Abstract: We present solutions to several problems concerning crossed products and tensor products of  $C^*$ - and von Neumann algebras. The common theme is our use of operator space techniques, in particular completely bounded module maps.

We prove that a locally compact group  $\mathcal{G}$  has the approximation property (AP) if and only if a non-commutative Fejér theorem holds for the associated  $C^*$ - or von Neumann crossed products. As applications, we answer three open problems. Specifically, we show that the AP always implies exactness. This generalizes a result of U. Haagerup and J. Kraus, and answers a question by K. Li. We also answer a problem of E. Bédos and R. Conti on discrete  $C^*$ -dynamical systems, and a question by M. Anoussis, A. Katavolos, and I.G. Todorov on bimodules over the group von Neumann algebra  $VN(\mathcal{G})$  for all locally compact groups  $\mathcal{G}$  with the AP. In our approach, we develop a notion of Fubini crossed product for locally compact groups, and a dynamical version of the AP for actions. (This is joint work with J. Crann.)

For a  $C^*$ -algebra  $\mathcal{A}$ , consider its projective (Banach) tensor square  $\mathcal{A} \otimes_\gamma \mathcal{A}$ ; if  $\mathcal{A}$  is commutative, this is the Varopoulos algebra  $V_{\mathcal{A}}$ . It has been open for almost 40 years to characterize when  $\mathcal{A} \otimes_\gamma \mathcal{A}$  is Arens regular. We solve this problem for arbitrary  $C^*$ -algebras, mainly relying on versions of the (commutative and non-commutative) Grothendieck Theorem. We show that Arens regularity of  $\mathcal{A} \otimes_\gamma \mathcal{A}$  is equivalent to  $\mathcal{A}$  having the Phillips property; hence, it is entirely encoded in the geometry of  $\mathcal{A}$ . The result generalizes prior work by A. Kumar, A.T.-M. Lau, M. Ljeskovac, V. Rajpal, A.M. Sinclair, and A. Ülger. In case  $\mathcal{A}$  is a von Neumann algebra, we obtain that  $\mathcal{A} \otimes_\gamma \mathcal{A}$  is Arens regular only if  $\mathcal{A}$  is finite-dimensional. We also show that this does not hold in general for non-selfadjoint dual (even commutative) operator algebras. For commutative  $C^*$ -algebras  $\mathcal{A}$ , we prove that the centre  $Z(V_{\mathcal{A}}^{**})$  is Banach algebra isomorphic to the extended Haagerup tensor product  $\mathcal{A}^{**} \otimes_{eh} \mathcal{A}^{**}$ .

Speaker: **Thomas Ransford** (Université Laval)

Title: *Jacobson's lemma, exponential spectrum and homotopy theory*

Abstract: Let  $G$  denote the group of invertible elements of a unital Banach algebra  $A$ . According to a well-known lemma of Jacobson, given  $a, b \in A$ , we have  $(1 - ab) \in G \iff (1 - ba) \in G$ .

Let  $G_1$  denote the connected component of  $G$  containing 1. Is it true that  $(1 - ab) \in G_1 \iff (1 - ba) \in G_1$ ? This problem dates back at least to 1992. I will present a solution based on ideas from homotopy theory, specifically, the Hopf fibration. (Joint work with Hubert Klaja.)

Speaker: **Volker Runde** (University of Alberta)

Title: *Wittstock moduli and their application to generalized notions of amenability*

Abstract: Let  $H$  be a Hilbert space, and let  $T : B(H) \rightarrow B(H)$  be a completely bounded map. As a consequence of Wittstock's operator valued Hahn–Banach theorem, there is a (not necessarily unique) completely positive operator  $|T| : B(H) \rightarrow B(H)$  with  $\||T|\|_{cb} \leq \|T\|$  such that  $|T| \pm \operatorname{Re} T$  and  $|T| \pm \operatorname{Im} T$  are completely positive. We explicitly describe  $|T|$  in the case where  $T$  is an elementary operator with coefficients in a  $C^*$ -algebra acting on  $H$  and use these moduli to shed light on the question whether or not an approximatively (or, equivalently, pseudo-) amenable  $C^*$ -algebra is nuclear.

Speaker: **Nico Spronk** (University of Waterloo)

Title: On operator amenability of Fourier-Stieltjes algebras

Abstract: Some of the most important algebras in harmonic analysis are the group and measure algebras,  $L^1(G)$  and  $M(G)$ , and their “Pontryagin duals” the Fourier and Fourier-Stieltjes algebras,  $A(G)$  and  $B(G)$ . I wish to survey the known amenability properties of these algebras, and discuss my recent work, characterizing operator amenability of  $B(G)$  for connected  $G$ .

Speaker: **Lyudmila Turowska** (Chalmers University of Technology and University of Gothenburg)

Title: *Operator-valued Schur multipliers and Herz-Schur multipliers of dynamical systems*

Abstract: Recently we generalized the notion of Schur multipliers to the  $C^*$ -algebra-valued case and extended the notion of Herz-Schur multipliers to the setting of non-commutative dynamical systems. We established their characterizations which generalize the classical descriptions of Schur multipliers and transference theorem that identify Herz-Schur multipliers with the invariant part of Schur multipliers. In this talk we shall show that like in the classical case, Herz-Schur multipliers of dynamical systems encode various approximation properties of the corresponding crossed product  $C^*$ -algebras. We shall also discuss special classes of the multipliers, in particular, (completely) compact and idempotents multipliers. The talk is based on a joint work with Andrew McKee, Adam Skalski and Ivan Todorov.

## Contributed Talks

Speaker **Ali Baklouti** (Université de Sfax, Tunisia)

Title: *Some Uncertainty principles on nilpotent Lie groups*

### In memory of Eberhard Kaniuth

Abstract: An old theorem of Hardy proved way back in 1933 says that a function  $f$  and its Fourier transform  $\hat{f}$  cannot both have arbitrary Gaussian decay unless  $f$  is identically zero. More precisely, if both  $f(x)e^{ax^2}$  and  $\hat{f}(\lambda)e^{b\lambda^2}$  are in  $L^\infty(\mathbb{R})$  for some  $a, b > 0$  then  $f = 0$  whenever  $ab > 1/4$ . Moreover, when  $ab = 1/4$  the function  $f$  is a constant multiple of  $e^{-ax^2}$  and when  $ab < 1/4$  there are infinitely many linearly independent functions satisfying both conditions. In 1982, Cowling and Price generalized Hardy's theorem by replacing the  $L^\infty$  estimates by  $L^p$  estimates. They have proved that if  $f(x)e^{ax^2} \in L^p(\mathbb{R})$  and  $\hat{f}(\lambda)e^{b\lambda^2} \in L^q(\mathbb{R})$  for some  $1 \leq p, q \leq \infty$ , then  $f = 0$  whenever  $ab > 1/4$ . The same conclusion holds even when  $ab = 1/4$  provided either  $p$  or  $q$  is finite.

In this talk, some analogues of Hardy and Cowling-Price principles for connected simply connected nilpotent Lie groups are formulated. This allows us to prove the sharpness of the constant  $\frac{1}{4}$  in the Hardy and in the Cowling-Price uncertainty principle for any nilpotent Lie group completing earlier works by E. Kaniuth and A. Kumar, A. Baklouti and S. Thangavelu, and A. Baklouti and E. Kaniuth. The orbit method and the Plancherel theory play an important role in the proof.

Speaker: **Ronalda Benjamin** (Stellenbosch University)

Title: *Connections between Fredholm theory and positivity in general ordered Banach algebras*

Abstract: Since its inception, Fredholm theory has become an important aspect of spectral theory. Among the spectra arising within Fredholm theory is the Weyl spectrum, which has been intensively studied by several authors, both in the operator case and in the general setting of Banach algebras.

Recall that the Weyl spectrum of a bounded linear operator  $T$  on a Banach space  $X$  is given by  $\bigcap_{K \in \mathcal{K}(X)} \sigma(T + K)$ , where  $\sigma(T)$  denotes the spectrum of  $T$  and  $\mathcal{K}(X)$  the closed ideal of all compact operators on  $X$ . In his 2009 paper, E. Alekhno showed that the Weyl spectrum of  $T$  on a Banach lattice  $E$  can be made more precise, and takes on the form  $\bigcap_{0 \leq K \in \mathcal{K}(E)} \sigma(T + K)$ . As is well-known, the algebra of all bounded linear operators on a Banach lattice is an important example of an ordered Banach algebra (OBA).

To investigate possible connections between Fredholm theory and positivity in arbitrary OBAs, we introduced the concept of 'upper Weyl spectrum' for an element of a general OBA. (The upper Weyl spectrum is generally larger than the well-known Weyl spectrum.)

In this talk results related to this new spectrum will be given. In particular, we shall discuss the relationship between the connected hulls of the Weyl and upper Weyl spectra.

Speaker: **Joel Feinstein** (University of Nottingham)

Title: *Regularity and non-regularity of  $R(X)$*

Abstract: Suppose that  $X$  and  $Y$  are compact plane sets such that  $R(X)$  and  $R(Y)$  are both regular. Does it follow that  $R(X \cup Y)$  is regular? We show that the answer is negative by constructing four compact plane sets  $X_i$  ( $i = 1, 2, 3, 4$ ) such that each  $R(X_i)$  is regular but  $R(\bigcup_{i=1}^4 X_i)$  is not regular.

This example is based on a previous example (F., 2001) of a compact plane set  $K$  with the property that  $R(K)$  is not regular, but  $R(K)$  has no non-trivial Jensen measures. The construction

of this set  $K$  was simplified recently in joint work with my former research student Hongfei Yang (F.-Yang, 2016).

Speaker: **Jorge Galindo** (IMAC, Universitat Jaume I, Spain)

Title: *Ergodic properties of convolution operators and on locally compact groups*

Abstract: A bounded operator  $T$  on a Banach space  $E$ ,  $T \in \mathcal{B}(E)$ , is said to be *mean ergodic* if the sequence of Cesàro means  $T_{[n]} = \frac{1}{n} \sum_{k=1}^n T^k$  is convergent in the strong operator topology.  $T$  is said to be *uniformly mean ergodic* if the sequence  $T_{[n]}$  is convergent in the operator norm.

For a locally compact group  $G$ , denote by  $\lambda_p: M(G) \rightarrow \mathcal{B}(L_p(G))$  the representation of the Banach algebra  $M(G)$ , given by the operators

$$\lambda_p(\mu)f = \mu * f, \quad f \in L_p(G).$$

In this talk we will address the problem of determining when the convolution operators  $\lambda_p(\mu)$  are mean ergodic or uniformly mean ergodic. It follows from general principles that, for  $1 < p < \infty$ ,  $\|\mu\| \leq 1$  is a sufficient condition for  $\lambda_p(\mu)$  to be mean ergodic,  $r(\lambda_p(\mu)) < 1$  is a sufficient condition for  $\lambda_p(\mu)$  to be uniformly mean ergodic and that a necessary condition for this latter property is that 1 is not an accumulation point of  $\sigma(\lambda_p(\mu))$ . We will find conditions on  $\mu$  and  $G$  under which these conditions turn into characterizations and examples showing that this is not always the case. We will also see that uniform mean ergodicity is in some cases equivalent to the stronger condition  $\|\mu\| < 1$ , for instance when the closed subgroup of  $G$  generated by the support of  $\mu$  is amenable but not compact.

This talk reports on ongoing joint work with Enrique Jordá (Universidad Politécnica de Valencia, Spain)

Speaker: **Olof Giselsson** (Chalmers University of Technology)

Title: The Shilov Boundary of the Quantum Matrix Ball

Abstract: The Shilov boundary of a compact Hausdorff space  $X$  relative to a uniform algebra  $\mathcal{A}$  in  $C(X)$  is the smallest closed subset  $K \subset X$  such that every function in  $\mathcal{A}$  achieves its maximum modulus on  $K$ . This notion is encountered, in particular, in the theory of analytic functions in relation to the maximum modulus principle. We will be interested in its non-commutative analog. The latter was introduced by W. Arveson.

In the middle of the 90s, within the framework of the quantum group theory, L.Vaksman and his coauthors started a “quantisation” of bounded symmetric domains. One of the simplest of such domains is the matrix ball  $\mathbb{D} = \{z \in Mat_m : zz^* < I\}$ , where  $Mat_m$  is the algebra of complex  $m \times m$  matrices.

The Shilov boundary of  $\mathbb{D}$  relative to the algebra of holomorphic functions in  $C(\overline{\mathbb{D}})$  is the set of unitary  $m \times m$ -matrices. In this talk I will discuss the Shilov boundary ideal for the  $q$ -analog of holomorphic functions on the unit ball. This is a joint work with O.Bershtein and L.Turowska.

Speaker: **Edmond Granirer** (University of British Columbia)

Title: *Some Geometric Properties of the Banach Algebras  $A_p^r(G) = A_p(G) \cap L^r(G)$*

Abstract: Let  $A_p(G)$  denote the Figa-Talamanca-Herz Banach algebra of the locally compact group  $G$ , thus  $A_2(G)$  is the Fourier algebra of  $G$ . If  $G$  is commutative then  $A_2(G) = L^1(\widehat{G})^\wedge$ . Let  $A_p^r(G) = A_p(G) \cap L^r(G)$  with norm  $\|u\|_{A_p^r} = \|u\|_{A_p} + \|u\|_{L^r}$ .

We investigate some Banach space geometric properties for these algebras (such as the Schur Property (SP), the Dunford-Pettis Property (DPP), the Krein-Milman Property (KMP)).

We ask which of these algebras possess the SP, the DPP or the KMP and give complete answers for weakly amenable  $G$  in terms of  $p$  and  $r$ .

Speaker: **Alexander Izzo** (Bowling Green State University)

Title: *Extensions of the notions of polynomial and rational hull*

Abstract: Extensions of the notions of polynomial and rational hull will be presented. Among the applications of these new hulls is a very flexible method of constructing polynomial and rational hulls without analytic structure.

Speaker: **Chi-Wai Leung** (The Chinese University of Hong Kong)

Title: *The hull-kernel topology on the Berkovich spectrum for commutative Banach rings*

Abstract: Let  $A$  be a commutative unital Banach ring. Let  $\mathcal{M}(A)$  be the Berkovich spectrum of  $A$ , ie., the set of all bounded multiplicative non-zero semi-norms defined on  $A$  which is endowed with the pointwise convergence topology.

Let  $C^b(T, k)$  be the Banach algebra of all bounded continuous  $k$ -valued functions defined on a zero-dimension topological space on  $T$ , where  $k$  is a complete valuation field. In this talk, we are going to investigate the regularity of  $C^b(T, k)$ . Also, from this we see that the Berkovich spectrum of  $C^b(T, k)$  is Hausdorff in the hull-kernel topology.

This is the joint work with Cheuk-Yin Lee.

Speaker: **Michael Lin** (Ben-Gurion University) **Jointly with Guy Cohen**

Title: *Joint and double coboundaries of transformations – an application of maximal spectral type of spectral measures*

Abstract: Let  $T$  be a bounded linear operator on a Banach space  $X$ ; the elements of  $(I - T)X$  are called *coboundaries*. For two commuting operators  $T$  and  $S$ , elements of  $(I - T)X \cap (I - S)X$  are called *joint coboundaries*, and those of  $(I - T)(I - S)X$  are *double coboundaries*. By commutativity, double coboundaries are joint ones. Are there any other?

Let  $\theta$  and  $\tau$  be commuting invertible measure preserving transformations of  $(\Omega, \Sigma, m)$ , with corresponding unitary operators induced on  $L_2(m)$ . We prove the existence of a joint coboundary  $g \in (I - U)L_2 \cap (I - V)L_2$  which is not in  $(I - U)(I - V)L_2$ .

For the proof, let  $E$  be the spectral measure on  $\mathbb{T}^2$  obtained by Stone's spectral theorem. Joint and double coboundaries are characterized using  $E$ , and properties of the maximal spectral type of  $E$ , together with a result of Foaiaş on multiplicative spectral measures acting on  $L_2$ , are used to show the existence of the required function.



Speaker: **Viktor Losert** (University of Vienna)

Title: *On weak amenability of Fourier algebras*

Abstract: For a connected Lie group  $G$  it was shown by Lee, Ludwig, Samei and Spronk that its Fourier algebra  $A(G)$  is weakly amenable only if  $G$  is abelian. We extend this result to general connected locally compact groups, extending an approach developed in special cases by Choi and Ghandehari.

Speaker: **Zinaida Lykova** (Newcastle University)

Title: *Hilbert space methods for the construction of analytic matrix functions*

Abstract: We study certain interpolation problems for analytic  $2 \times 2$  matrix-valued functions on the unit disc  $\mathbb{D}$ . We obtain a new solvability criterion for two such problems, special cases of the  $\mu$ -synthesis problem. For certain domains  $\mathcal{X}$  in  $\mathbb{C}^2$  and  $\mathbb{C}^3$  we describe a rich structure of interconnections between four objects: the set  $\text{Hol}(\mathbb{D}, \mathcal{X})$  of analytic functions from the disc into  $\mathcal{X}$ , the  $2 \times 2$  matricial Schur class  $\mathcal{S}^{2 \times 2}$ , the Schur class  $\mathcal{S}_2$  of the bidisc, and the set  $\mathcal{R}$  of pairs of positive kernels on the bidisc subject to a boundedness condition. The rich structure related to the construction of analytic matrix functions can be summarised diagrammatically as

$$\begin{array}{ccc} \mathcal{S}^{2 \times 2} & \longleftrightarrow & \mathcal{R} \\ \updownarrow & & \updownarrow \\ \text{Hol}(\mathbb{D}, \mathcal{X}) & \longleftrightarrow & \mathcal{S}_2. \end{array}$$

This rich structure combines with the classical realization formula and Hilbert space models in the sense of Agler to give an effective method for the construction of the required interpolating functions.

The talk is based on the papers

J. Agler, Z. A. Lykova and N. J. Young, A case of  $\mu$ -synthesis as a quadratic semidefinite program, *SIAM J. Control and Optimization*, **51** (3) (2013) 2472-2508.

D. C. Brown, Z. A. Lykova and N. J. Young, A rich structure related to the construction of holomorphic matrix functions, *Journal of Functional Analysis*, **272**(4) (2017), 1704–1754.

Speaker: **Nicholas Manor** (University of Waterloo)

Title: Exactness vs  $C^*$ -exactness for certain non-discrete groups

Abstract: It is known that exactness for a discrete group  $G$  is equivalent to  $C^*$ -exactness, i.e., the exactness of the reduced  $C^*$ -algebra  $C_r^*(G)$ . It is a major open problem to determine whether this equivalence holds for all locally compact groups, but the problem has recently been reduced by Cave and Zacharias to the case of totally disconnected (td) unimodular groups. We will discuss ways to extend the equivalence of exactness and  $C^*$ -exactness to classes of non-discrete groups. These include the td groups admitting an invariant neighbourhood of the identity, and a family of td unimodular groups introduced by Yuhei Suzuki in the context of  $C^*$ -simplicity.

Speaker: **Javad Mashreghi** (Université Laval)

Title: *Polynomial approximation in Banach spaces of analytic functions*

Abstract: Let  $X$  be a Banach holomorphic function space on the unit disk. A linear polynomial approximation scheme for  $X$  is a sequence of bounded linear operators  $T_n : X \rightarrow X$  with the property that, for each  $f \in X$ , the functions  $T_n(f)$  are polynomials converging to  $f$  in the norm of the space. We completely characterize those spaces  $X$  that admit a linear polynomial approximation scheme. In particular, we show that it is not sufficient merely that polynomials be dense in  $X$ .

Joint work with T. Ransford.

Speaker: **Frédéric Morneau-Guérin** (Université Laval)

Title: *Stability of the space of square-summable sequences with respect to convolution*

Abstract: It is widely known that the weighted  $L_p$ -space on a locally compact topological group  $G$  is stable with respect to convolution if the weight function  $w$  is weakly sub-convolutive. But there are numerous examples showing that this sufficient condition is not necessary. There is, however, a type of group, i.e. the discrete abelian groups, for which there remains a possibility that weak sub-convolutivity truly characterizes those weights entailing the stability of the  $L_p$ -space of functions with respect to convolution. In this talk, we will reinterpret this question (in the particular case of  $p = 2$ ) in light of the theory of reproducing kernel Hilbert spaces and in the context of the operator theory.

Speaker: **Sonja Mouton** (Stellenbosch University)

Title: *Linking the boundary and exponential spectra via the restricted topology*

Abstract: If  $a$  is an element of a complex Banach algebra  $A$  with unit 1, then the boundary spectrum  $S_\partial(a) = \{\lambda \in \mathbb{C} : a - \lambda 1 \in \partial A^{-1}\}$  of  $a$  (see [2]) is a compact set in  $\mathbb{C}$  that lies between the usual topological boundary of the spectrum and the spectrum  $\sigma(a)$  itself, i.e.

$$\partial\sigma(a) \subseteq S_\partial(a) \subseteq \sigma(a).$$

It therefore seems natural to view  $\partial\sigma(a)$ , on the one hand, and  $S_\partial(a)$ , on the other hand, as the “thin” and “fat” boundaries of  $\sigma(a)$ , respectively. We will show that, using closed subalgebras  $B$ , it is possible to define a topology on  $A$  (different from the norm-topology) in such a way that a whole range of “boundaries” can be obtained, with  $B = \mathbb{C}$  giving the “thin” boundary and  $B = A$  the “fat” boundary of  $\sigma(a)$ . Recalling a certain duality between the boundary  $\partial\sigma(a)$  and the connected hull  $\eta\sigma(a)$  of the spectrum of  $a$  and bearing in mind the fact that the exponential spectrum  $\varepsilon(a)$  of  $a$  (see [1]) lies between  $\sigma(a)$  and  $\eta\sigma(a)$ , i.e.

$$\sigma(a) \subseteq \varepsilon(a) \subseteq \eta\sigma(a),$$

we also investigate the “connected hulls” accompanying these new “boundaries”. This is joint work with Robin Harte (see [3]).

## References

- [1] R. E. Harte: The exponential spectrum in Banach algebras. *Proc. Amer. Math. Soc.* 58, 114–118 (1976).
- [2] S. Mouton: On the boundary spectrum in Banach algebras. *Bull. Austral. Math. Soc.* 74, 239–246 (2006).
- [3] S. Mouton and R. Harte: Linking the boundary and exponential spectra via the restricted topology. *J. Math. Anal. Appl.* 454, 730–745 (2017).

Speaker: **Chi-Keung Ng** (Chern Institute of Mathematics, Nankai University)

Title: *Normal states are determined by their facial distances*

Abstract: Let  $M$  be a semi-finite  $W^*$ -algebra with normal state space  $\mathfrak{S}(M)$ . For any  $\phi \in \mathfrak{S}(M)$ , let  $M_\phi := \{x \in M : x\phi = \phi x\}$  be the centralizer of  $\phi$  with center  $\mathcal{Z}(M_\phi)$ . We show that for  $\phi, \psi \in \mathfrak{S}(M)$ , the following are equivalent.

- $\phi = \psi$ .
- $\mathcal{Z}(M_\psi) \subseteq \mathcal{Z}(M_\phi)$  and  $\phi|_{\mathcal{Z}(M_\phi)} = \psi|_{\mathcal{Z}(M_\phi)}$ .
- $\phi, \psi$  have the same distances to all the closed faces of  $\mathfrak{S}(M)$ .

We are then able to give an alternative proof of the fact that metric preserving surjections between normal state spaces of semi-finite  $W^*$ -algebras are induced by Jordan  $*$ -isomorphisms between the underlying algebras. Applications to  $F$ -algebras (in particular, Fourier and Fourier-Stieltjes algebras of locally compact quantum groups) are provided.

This is joint work with A.T.-M. Lau and N.C. Wong.

Speaker: **Przemysław Ochrisko** (Chalmers University of Technology and the University of Gothenburg)

Title: *Inversion problem in measure algebras and Fourier-Stieltjes algebras*

Abstract: Let  $G$  be a locally compact Abelian group with its dual  $\widehat{G}$  and let  $M(G)$  denote the Banach algebra of complex-valued measures on  $G$ . The classical Wiener-Pitt phenomenon asserts that the spectrum of a measure may be strictly larger than the closure of the range of its Fourier-Stieltjes transform. In particular, if  $G$  is non-discrete, there exists  $\mu \in M(G)$  such that  $|\widehat{\mu}(\gamma)| > c > 0$  for every  $\gamma \in \widehat{G}$  but  $\mu$  is not invertible. In the paper [N], N. Nikolski suggested the following problem.

**Problem** *Let  $\mu \in M(G)$  satisfy  $\|\mu\| \leq 1$  and  $|\widehat{\mu}(\gamma)| \geq \delta$  for every  $\gamma \in \widehat{G}$ . What is the minimal value of  $\delta_0$  assuring the invertibility of  $\mu$  for every  $\delta > \delta_0$ ? What can be said about the inverse (in terms of  $\delta$ )?*

In my talk I show that  $\delta_0 = \frac{1}{2}$  is the optimal value for the first question (for non-discrete  $G$ ). Also, I will present a partial solution for the quantitative variant of the problem (second question): if all elements of  $G$  (except the unit) are of infinite order then we can control the norm of the inverse for every  $\delta > \frac{-1+\sqrt{33}}{8} \simeq 0,593$ . This improves the original result of Nikolski:  $\delta > \frac{1}{\sqrt{2}} \simeq 0,707$ .

The second part of my presentation will be devoted to the study of an analogous problem for Fourier-Stieltjes algebras. In particular, I will show that  $\delta_0 = \frac{1}{2}$  is again the optimal value and that the result of Nikolski on  $\delta > \frac{1}{\sqrt{2}}$  holds true in this setting too.

The talk is based on a preprint [OW] written in collaboration with Mateusz Wasilewski.

## References

- [OW] P. Ochrisko, M. Wasilewski: *Inversion problem in measure and Fourier-Stieltjes algebras*, preprint available on arxiv.org with the identifier: 1808.06225.
- [N] N. Nikolski: *In search of the invisible spectrum*, Ann. Inst. Fourier (Grenoble), vol. 49, no. 6, pp. 1925–1998, 1999.

Speaker: **Irina Peterburgsky** (Suffolk University, Boston)

Title: *Extremal functions for the norms of operators acting into complex Banach spaces*

Abstract: We consider extremal problems on norms of a wide class of linear operators on the spaces of bounded analytic functions of the complex variable acting into complex Banach spaces. For extremal problems under consideration, significantly new facts in comparison with extremal problems for classes of scalar-valued bounded analytic functions, are the relationships between the geometry of the unit ball of the Banach space of function values and the properties of extremal functions (their existence, uniqueness, characteristic properties, structure). The purpose of this paper is to study these relationships.

Speaker: **G. Racher** (Salzburg, Austria)

Title: *On Fourier multipliers*

Abstract: We show that any locally compact group  $G$  containing an open abelian subgroup admits a nonzero bounded linear map from its von Neumann algebra into  $L^2(G)$  which intertwines the respective actions by its Fourier algebra.

Speaker: **Piotr Soltan** (University of Warsaw)

Title: *Quantum families of maps*

Abstract: Quantum families of maps are non-commutative spaces which generalize spaces of continuous maps between topological spaces. They are interesting examples of non-commutative spaces and often carry additional structure which makes them quantum semigroups or quantum groups. Examples of such objects crop up in various contexts including so called "synchronous games" and "quantum strategies". I will present a number of examples of such non-commutative spaces and show that while sometimes they behave in ways similar to classical spaces of maps, in other situations they exhibit rather unexpected properties.

Speaker: **Aasaimani Thamizhazhagan** (University of Waterloo)

Title: *On the structure of invertible elements in Fourier-Stieltjes algebras*

Abstract: For a locally compact abelian group  $G$ , J.L. Taylor(1971) gave a complete characterization of invertible elements in its measure algebra  $M(G)$  engaging semigroup theory and cohomological calculations. Via Fourier-Stieltjes transforms, this characterization can be done in the context of Fourier-Stieltjes algebras  $B(G)$  of abelian  $G$ . Following the investigation begun in M.E. Walter's work (1975), we have established this latter characterization for Fourier-Stieltjes algebra  $B(G)$  of certain class of locally compact groups in particular, many totally minimal groups and  $ax + b$  group.

Speaker: **Hans-Olav Tylli** (University of Helsinki)

Title: *Structure of the compact-by-approximable quotient algebra for Banach spaces failing the approximation property*

Abstract: I will describe joint work with Henrik Wirzenius (Helsinki) on the structure of the quotient algebra  $\mathfrak{A}_X = \mathcal{K}(X)/\mathcal{A}(X)$  of the compact-by-approximable operators on Banach spaces  $X$  failing the approximation property. Here  $\mathcal{K}(X)$  is the algebra of compact operators on  $X$  and  $\mathcal{A}(X) = \overline{\mathcal{F}(X)}$  the closure of the finite rank operators. The radical Banach algebra  $\mathfrak{A}_X$  is poorly understood, and our work is motivated by questions of Dales.

I will concentrate on problems about the size of  $\mathfrak{A}_X$ . For instance, we show that there is a closed linear subspace  $X \subset \ell^p$  for  $1 \leq p < \infty$  and  $p \neq 2$ , or  $X \subset c_0$ , such that  $c_0$  embeds isomorphically into  $\mathfrak{A}_X$ . Moreover,  $c_0$  also embeds isomorphically into  $\mathfrak{A}_Z$  for the class of Banach

spaces  $Z$  constructed by Willis, where  $Z$  fails the approximation property but has the bounded compact approximation property. The talk is based on the preprint *arXiv:1811.09402v1 [math.FA]*

Speaker: **Henrik Wirzenius** (University of Helsinki)

Title: *Algebraic properties of the Banach algebra of compact-by-approximable operators*

Abstract: Let  $\mathcal{K}(X)$  denote the Banach algebra of compact operators acting on a Banach space  $X$  and  $\mathcal{A}(X) = \overline{\mathcal{F}(X)}$  the approximable operators  $X \rightarrow X$ . In this talk, we will discuss recent observations about algebraic properties of the quotient algebra  $\mathcal{K}(X)/\mathcal{A}(X)$  of compact-by-approximable operators and present examples where  $\mathcal{K}(X)/\mathcal{A}(X)$  contains non-trivial closed ideals. This is a joint work with Hans-Olav Tylli (University of Helsinki).

Speaker: **Jared White** (Université de Franche-Comté)

Title: *Measure algebras on locally compact groups and their left ideals*

Abstract: We shall discuss the left ideal structure of measure algebras on locally compact groups. The talk will mostly focus on the case in which the underlying group is Hermitian, in which case there is a known classification of the maximal modular left ideals of the group algebra in terms of the irreducible representations of the group, which will be our starting point. We shall present new results that concern when maximal left ideals in measure algebras are weak\*-closed or finitely-generated.

Speaker: **Marshall Whittlesey** (Cal State San Marcos)

Title: *Polynomial hulls containing higher orders of analytic structure*

Abstract: For a compact set  $K \subset \mathbb{C}^n$ , the polynomial convex hull of  $K$  is the set of all  $w \in \mathbb{C}^n$  such that for every polynomial  $P(z)$ ,  $|P(w)|$  is less than or equal to the maximum modulus of  $P(z)$  on  $K$ . The polynomial convex hull is the maximal ideal space of the uniform algebra  $P(K)$  of continuous complex-valued functions which are uniform limits of polynomials on  $K$ . A typical problem is to try to decide whether the polynomial convex hull of a set is composed of 1-dimensional *analytic structure*, i.e., analytic varieties with boundary in  $K$ . In this talk we discuss conditions where higher dimensional analytic structure can be found. Let  $S^{n-1}$  be the unit sphere in  $\mathbb{C}^n$ , and  $K \subset S^{n-1} \times \mathbb{C}$ . Then we examine when the polynomial convex hull of  $K$  contains graphs of analytic functions over the ball.

Speaker: **Matthew Wiersma** (University of California San Diego)

Title: *Hermitian groups are amenable*

Abstract: A locally compact group  $G$  is *Hermitian* if the spectrum  $\sigma_{L^1(G)}(f)$  is contained in  $\mathbb{R}$  for every  $f = f^* \in L^1(G)$ . Examples of Hermitian groups include all abelian locally compact groups. A question from the 1960s asks whether every Hermitian group is amenable. I will speak on the history and recent affirmative solution to this problem.