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EARNINGS MULTIFACTOR PROCESS, RESIDUAL INCOME VALUATION, AND LONG-RUN RISK

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ABSTRACT

In this paper, we extend the residual income valuation model by incorporating the long-run sensitivity of earnings to various economic factors. Our valuation procedure integrates the multidimensionality of uncertainty, as well as the long-run concept of risk (recently proposed in finance and accounting). Our extension model begins with an earnings multifactor process, uses an intertemporal equilibrium version of the residual income valuation method, and sums over many periods. In this manner, we demonstrate that the abnormal earnings growth rate of a firm is linearly and positively related to *N* sensitivity coefficients, given by the long-run sensitivity between abnormal earnings and economic factors. We then reveal that the corresponding equity value of the firm is a function of the current book value, abnormal earnings, and *N* long-run risk parameters. In the context of the residual income valuation approach, these findings suggest that earnings sensitivity to several factors represents an additional technique to estimate risk (in the long run).

INTRODUCTION

The Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) indicates that the expected return of an asset is linearly related to a single measure of risk, the market beta, obtained from the covariance between asset returns and market returns. Multifactor pricing models are more general. They assume that asset returns are generated by several factors and demonstrate that the expected return of an asset is linearly related to several risk measures.

Multifactor pricing models were initiated by Merton (1973) via the Intertemporal-CAPM and by Ross (1976) via Arbitrage Pricing Theory (APT). In the intertemporal framework of Merton, the market portfolio serves as one factor, and state variables serve as additional factors. These additional factors arise from the investor's demand to hedge against uncertainty in future investment opportunities. As a result, the Intertemporal-CAPM shows that the expected excess return on any asset is given by a multi-beta version of the CAPM, where the number of betas is equal to one plus the number of state variables. The APT assumes that the return on any asset is generated by different economic factors. Given this return-generating process, Ross demonstrates that the absence of arbitrage implies that the expected asset return is a function of the asset's sensitivities to factors. Following these fundamental theories, Fama and French (1993) propose a three-factor model. The model was based on previous observations that demonstrate an empirical relationship between stock returns, size, and book-to-market equity. More precisely, the three-factor model supposes that the excess returns of an asset are generated by the following factors: (1) the excess return of the market portfolio; (2) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big); and (3) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to market stocks (HML, high minus low). Therefore, the model reveals that the expected excess return of an asset is related to three factor sensitivities (three betas).

As noted by Campbell (2000, p. 1525), the vast available literature on multifactor models can be understood through the structure of the stochastic discount factor.[1] More precisely, if there are several common factors that influence undiversifiable risk, and if we accept the assumption that the stochastic discount factor is a linear combination of K common factors, then a multifactor pricing model holds. Cochrane (1996), Jagannathan and Wang (1996), and Lettau and Ludvigson (2001), implement this approach. Many empirical works since have further confirmed the importance of a multifactor approach, notably Pàstor and Stambaugh (2002), Lawrence et al. (2007), Hou et al. (2011), and Fama and French (2012; 2015; 2016; and 2017). As an example, Fama and French (2015) add *profitability* and *investment* factors to the *market*, *size*, and *book-to-market* factors of the original model.

In this paper, we adopt a multifactor approach and develop a theoretical extension of the residual income valuation model that integrates the long-run sensitivity of earnings to various economic factors.

[1] The term *stochastic discount factor* simply indicates that the price of an asset can be estimated by discounting the future cash flow by a *stochastic* (or uncertain) factor.

As mentioned by Bansal et al. (2016), the long-run concept of risk initiated by Bansal and Yaron (2004) has motivated a significant amount of research in macro and financial economics. Indeed, Bansal and Yaron (2004) maintain that consumption and dividend growth rates include a small long-run component that can explain key asset market phenomena. They indicate that long-run risk in cash flow should carry higher risk compensation and explain differences in asset expected returns. Moreover, Bansal et al. (2005) show that long-run covariance between dividends and aggregate consumption accounts for more than 60% of the cross-sectional variation in risk premia. In addition, Hansan et al. (2008) demonstrate that growth-rate variations in consumption and cash flows have important consequences for asset valuation. Furthermore, Da (2009) reveals that the long-run covariance between earnings and aggregate consumption explains more than 56% of the cross-sectional variation in risk premia. Also, Bansal and Kiku (2011) suggest that the long-run equilibrium relation measured via a stochastic cointegration between aggregate consumption and dividends has significant implications for dividend growth rates and returns dynamics. Additionally, Bergeron (2013a, and 2013b) derives a theoretical stock valuation method that takes into account the long-run concept of risk, estimated with dividends. Further, Bansal et al. (2016) point out the importance of time aggregation for estimating the dynamics of long-run risks. More recently, Bergeron et al. (2018) integrates the concept of long-run risk into the residual income valuation model, using an intertemporal framework.

However, none of the above-mentioned studies proposes a theoretical extension of the residual income model that integrates both the multidimensionality of uncertainty and the long-run concept of risk.

The residual income valuation model expresses a company's fundamental value as the sum of its book value and the present value of its future residual income.[2] This approach began with Edwards and Bell (1961) and Peasnell (1981, 1982), and was popularized by numerous researchers, notably

[2] Residual income represents the economic profit of the business after deducting the cost of capital. Here, the terms *residual income* and *abnormal earnings* are interchangeable.

Ohlson (1995), Feltham and Ohlson (1999), Baginski and Wahlen (2003), Ohlson and Juettner-Nauroth (2005), Nekrasov and Shroff (2009), Beynon and Clatworthy (2013), and Hand et al. (2017).

The development of our model is based on the same intertemporal equilibrium framework used by Bergeron et al. (2018), to which we add a multifactor generating process. First, we assume that *earnings growth rate*s or *abnormal earnings growth rates* are generated by several economic factors. Then, we express the residual income valuation model in an intertemporal context. More precisely, we express a company's fundamental value as the sum of its book value and the present value of its future residual income, where future cash flows are discounted by a stochastic discount factor equivalent to the consumption marginal rate of substitution. Next, we derive the residual income growth rate of the firm for a single period, under equilibrium conditions. Finally, we sum over several periods and isolate the corresponding equity value.

In this manner, we demonstrate that the abnormal earnings growth rate of a firm is linearly and positively related to *N* sensitivity coefficients, given by the long-run sensitivity between earnings and economic factors. Thus, our main result reveals that the intrinsic equity value of a firm is a function of its current book value, abnormal earnings, and *N* long-run risk parameters.

Our methodology for this paper differs from Bergeron et al. (2018) in three significant ways. First, the multidimensionality of uncertainty is added, using a multifactor generating process. Second, the constant relative risk aversion assumption via the power utility function is relaxed. Third, the normality assumption and Taylor series approximation are not required.

The remainder of this paper is organized in four sections. The following section presents the earnings multifactor process. The second section describe the residual income valuation model in an intertemporal context. The third section integrate the multiple dimensions of long-run risk into the equity valuation process. The fourth and final section concludes the paper.

THE EARNINGS MULTIFACTOR PROCESS

Following Bergeron (2013b, p. 3), the primary assumption of our multifactor model is that *earnings growth rate*s or *abnormal earnings growth rates* are generated by several factors. The abnormal earnings (or the residual income) of firm i, at time $t+1$, $\tilde{X}^{a}_{i,t+1}$, is calculated in this standard manner: $\tilde{X}^{a}_{i,t+1} = \tilde{X}_{i,t+1} - R_{F,t+1}Y_{it}$, where $\tilde{X}_{i,t+1}$ represents the random earnings of firm i, at time $t+1$, Y_{it} is the book value of firm i, at time t, and $R_{F,t+1}$ equals the risk-free rate of return between time t and $t+1$.[3] Also, the abnormal earnings growth rate of firm i, between time t and $t+1$, $\tilde{g}^{a}_{i,t+1}$, is assumed to be a linear function of N economic factors as shown below:

$$\tilde{g}^{a}_{i,t+1} = \alpha_{it} + \beta_{1it}\tilde{F}_{1,t+1} + \beta_{2it}\tilde{F}_{2,t+1} + \cdots + \beta_{Nit}\tilde{F}_{N,t+1} + \tilde{\varepsilon}_{i,t+1}, \tag{1a}$$

with,

$$E_t\left[\tilde{\varepsilon}_{i,t+1}\right] = COV_t\left[\tilde{\varepsilon}_{i,t+1},\ \bullet\right] = 0,$$

where α_{it} is the growth rate intercept for firm i at time t, $\tilde{F}_{k,t+1}$ is the factor k at time $t+1$, β_{kit} is the earnings growth rate sensitivity to factor k for firm i at time t, and $\tilde{\varepsilon}_{i,t+1}$ is the usual random term for firm i at time $t+1$ ($k = 1, 2, 3, \ldots, N$; $i = 1, 2, 3, \ldots, M$; $t = 0, 1, 2, \ldots, \infty$).[4] To simplify the notation, we use matrix algebra to rewrite the multifactor process in this compact form:

$$\tilde{g}^{a}_{i,t+1} = \alpha_{it} + \boldsymbol{\beta}'_{it}\tilde{\boldsymbol{F}}_{t+1} + \tilde{\varepsilon}_{i,t+1}, \tag{1b}$$

where $\tilde{\boldsymbol{F}}_{t+1}$ is a *column vector* containing the elements $\tilde{F}_{1,t+1}, \tilde{F}_{2,t+1}, \ldots, \tilde{F}_{N,t+1}$, while $\boldsymbol{\beta}'_{it}$ is a *row vector* containing the elements $\beta_{1it}, \beta_{2it}, \ldots, \beta_{Nit}$.

As the standard multifactor model for returns, the earnings process expressed by Equation (1) represents an approximation of reality, and the factors to be integrated into the model are not

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[3] Here, the tilde (~) denotes a random variable. Operators E_t, VAR_t, and COV_t refer respectively to mathematical expectations, variance, and covariance, where index t indicates that we consider the available information at time t.

[4] The second line of Equation (1a) simply assumes that the expected value of the usual random term is zero, as the covariance between this random term and any other variables.

determined by any economic theory. If the number of factors equals one, and if this factor corresponds to the aggregate consumption growth, then the only sensitive coefficient (β) is obtained from the covariance between the firm's earnings and aggregate consumption, as in Da (2009). If this lone factor corresponds to global market earnings, then the sensitive coefficient is similar to the well-known accounting beta. Finally, if the number of factors is greater than one, then the above process reveals the multidimensionality of systematic earnings risk, for equity valuation.

THE RESIDUAL INCOME VALUATION MODEL

The residual income valuation model stipulates that equity value can be split into two components: the current book value and the present value of future discounted cash flows not captured by the current book value (*residual income*). This model is based on the fundamental dividend discount formula, and the clean surplus relation. More precisely, assuming a restrictive intertemporal economy in which the representative agent maximizes its time-separable utility function, the residual income valuation model expresses the equilibrium equity market value of firm i, at time t, V_{it}, in the following manner (see Bergeron, 2018, p. 7):[5]

$$V_{it} = Y_{it} + E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \tilde{X}^a_{i,t+s}, \tag{2}$$

where δ equals the time discount factor, U' is the derivative of the utility function, C_t represents consumption at time t, $\tilde{C}_{t+s}$ denotes consumption at time $t+s$, and $\tilde{X}^a_{i,t+s}$ corresponds to the abnormal earnings of firm i, at time $t+s$ ($s = 1, 2, 3, \dots, \infty$). Given the available information at time t, Equation (2) indicates that the equilibrium equity market value of a firm is equal to its book value plus the expected present value of all future abnormal earnings where the stochastic discount factor is equivalent to $\delta^s U'(\tilde{C}_{t+s})/\, U'(C_t)$. Contrary to the framework adopted in many other intertemporal models,[6]the present definition of the stochastic discount factor is note based on the standard assumption of a *constant relative risk aversion* via a power utility function. This allows us to generalize the valuation process and reduce the number of restrictive assumptions.

[5] See also Nekrasov and Shroff (2009, p. 1987).

[6] See, for example, Bansal and Kiku (2011) or Bergeron et al. (2018).

Equation (2) also indicates that the difference between the equity value of the firm and its book value corresponds to the present value of all future abnormal earnings, as shown below:

$$S_{it} = E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \tilde{X}^a_{i,t+s}, \tag{3}$$

where S_{it} is such that: $S_{it} \equiv V_{it} - Y_{it}$. Since the current abnormal earnings of firm *i* at time *t*, X^a_{it}, are known given the available information, we can write:

$$S_{it} = X^a_{it} E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \frac{\tilde{X}^a_{i,t+s}}{X^a_{it}}, \tag{4}$$

or, to simplify the notation:

$$S_{it} = X^a_{it} E_t[\tilde{\chi}_{it}], \tag{5}$$

where the random variable $\tilde{\chi}_{it}$ represents the sum of discounted abnormal earnings growth, defined in this manner: $\tilde{\chi}_{it} \equiv \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \frac{\tilde{X}^a_{i,t+s}}{X^a_{it}}$. If the sequence of variable $\tilde{\chi}_{it}$ is independent and identically distributed (*i.i.d.*), then $\tilde{\chi}_{it} = \tilde{\chi}_i$ (for $t = 0, 1, 2, \dots$), which gives us:

$$S_{it} = X^a_{it} E_t[\tilde{\chi}_i]. \tag{6}$$

Taking the expected value on each side of Equation (6), we get:

$$S_{it} = X^a_{it} E[\tilde{\chi}_i] = X^a_{it} \rho_i, \tag{7}$$

with $\rho_i \equiv E[\tilde{\chi}_i]$. Therefore, given the available information at time *t*, we can establish that the stochastic difference between the market equity value of firm *i*, at time $t + 1$, $\tilde{V}_{i,t+1}$, and the corresponding book value ($\tilde{Y}_{i,t+1}$), is directly proportional to the next abnormal earnings, that is:

$$\tilde{S}_{i,t+1} = \tilde{X}^{a}_{i,t+1}\rho_i, \tag{8}$$

where $\tilde{S}_{i,t+1}$ is such that: $\tilde{S}_{i,t+1} \equiv \tilde{V}_{i,t+1} - \tilde{Y}_{i,t+1}$. In brief, as in Bergeron et al. (2018), but without the restrictive assumption of a power utility function, we can easily see that the equity value of a firm and the abnormal earnings are stochastically cointegrated.

EQUITY VALUE AND THE MULTIPLE DIMENSIONS OF LONG-RUN RISK

In this section, we demonstrate that the multiple dimensions of long-run risk determine the intrinsic equity value of a firm, in addition to its book value and abnormal earnings. We begin by isolating the expected abnormal earnings growth rate of a firm for one period and one factor. Then, we express the expected value for one period and several factors. Thereafter, we express the expected value for many periods and several factors. Finally, we integrate the current book value of the firm to determine its intrinsic equity value. Our development is similar to any multifactor model that, given the *N* factors process, deduces asset values from equilibrium conditions.[7]

Earnings growth with one period and one factor

In Equation (3), the stochastic discount factors corresponds to the marginal rate of substitution between consumption at time t and $t+s$, calculated in this manner: $\widetilde{M}_{t+s} = \delta^{s}U'(\tilde{C}_{t+s})/U'(C_t)$, for $s = 1, 2, 3, \dots, \infty$. Recursively, Equation (3) can also be expressed for one period in the following manner:[8]

$$S_{it} = E_t\left[\widetilde{M}_{t+1}\left(\tilde{S}_{i,t+1} + \tilde{X}^{a}_{i,t+1}\right)\right], \tag{9}$$

where $\widetilde{M}_{t+1}$ corresponds to the marginal rate of substitution between time t and $t+1$. This equation is similar (in form) to the basic equation of asset pricing, for a single period.[9] Introducing Equation (7) and (8) into Equation (9) indicates that:

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[7] See, for example, the APT of Ross (1976), or the intertemporal-CAPM of Merton (1973). See also, Campbell (2000, p. 1525), or Bergeron (2013b), among others.
[8] See Bergeron et al. (2018) or Huang and Litzenberger (1988, p. 202).
[9] See, for example, Campbell (2000, p. 1517).

$$X_{it}^{a}\rho_i = E_t\left[\widetilde{M}_{t+1}\left(\tilde{X}_{i,t+1}^{a}\rho_i + \tilde{X}_{i,t+1}^{a}\right)\right]. \tag{10}$$

After simple manipulations, we can write:

$$1 = E_t\left[\widetilde{M}_{t+1}(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1})\right], \tag{11a}$$

knowing that: $\tilde{g}_{i,t+1}^{a} \equiv \tilde{X}_{i,t+1}^{a}/X_{it}^{a} - 1$. Taking the expected value on each side of Equation (11a) allows us to release the index t of the conditional operator, to show that:

$$1 = E\left[\widetilde{M}_{t+1}(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1})\right]. \tag{11b}$$

Multiplying by $E[\widetilde{M}_{t+1}]$ on each side of Equation (11b) implies that:

$$E\left[\widetilde{M}_{t+1}\right]/E\left[\widetilde{M}_{t+1}\right] = E\left[\widetilde{M}_{t+1}(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1})\right]. \tag{12}$$

Subtracting the left-hand-side of the equation from both sides, we have:

$$0 = E\left[\widetilde{M}_{t+1}(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1})\right] - E\left[\widetilde{M}_{t+1}\right]/E\left[\widetilde{M}_{t+1}\right]. \tag{13}$$

Integrating the last element into the expectation operator and simplifying, yields:

$$0 = E\left[\widetilde{M}_{t+1}\left\{(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1}) - 1/E\left[\widetilde{M}_{t+1}\right]\right\}\right]. \tag{14}$$

Using the mathematical definition of covariance, Equation (14) reveals that:

$$\begin{aligned} &COV\left[\widetilde{M}_{t+1}, (1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1}) - 1/E\left[\widetilde{M}_{t+1}\right]\right] \\ &= -E\left[\widetilde{M}_{t+1}\right] E\left[(1 + \tilde{g}_{i,t+1}^{a})(1 + \rho_i^{-1}) - 1/E\left[\widetilde{M}_{t+1}\right]\right]. \end{aligned} \tag{15}$$

Utilizing the basic mathematical properties of covariance, and rearranging, shows that:

$$COV\left[\widetilde{M}_{t+1}, \tilde{g}_{i,t+1}^{a}\right](1 + \rho_i^{-1}) = 1 - E\left[\widetilde{M}_{t+1}\right]E\left[1 + \tilde{g}_{i,t+1}^{a}\right](1 + \rho_i^{-1}). \tag{16}$$

Isolating the expected abnormal earnings growth rate of the firm thus indicates that:

$$E\left[1+\tilde{g}_{i,t+1}^{a}\right]=1/E\left[\widetilde{M}_{t+1}\right](1+\rho_i^{-1})-COV\left[\widetilde{M}_{t+1},\tilde{g}_{i,t+1}^{a}\right]/E\left[\widetilde{M}_{t+1}\right]. \tag{17}$$

Equation (17) represents the expected abnormal earnings growth rate of a firm, for one period, under equilibrium conditions. Here, the only factor that is assumed to be correlated with the earnings growth rate is the consumption marginal rate of substitution (*M*).

Earnings growth with one period and many factors

To introduce the multidimensionality of earnings risk, we integrate into Equation (17) the multifactor model formulated by Equation (1b), in the following manner:

$$E\left[1+\tilde{g}_{i,t+1}^{a}\right]=1/E\left[\widetilde{M}_{t+1}\right](1+\rho_i^{-1})$$
$$-COV\left[\widetilde{M}_{t+1},\alpha_{it}+\boldsymbol{\beta}'_{it}\widetilde{\boldsymbol{F}}_{t+1}+\tilde{\varepsilon}_{i,t+1}\right]/E\left[\widetilde{M}_{t+1}\right]. \tag{18}$$

Using simple mathematical covariance properties, and taking into account the basic definition of the usual random term, we can write:

$$E\left[1+\tilde{g}_{i,t+1}^{a}\right]=1/E\left[\widetilde{M}_{t+1}\right](1+\rho_i^{-1})-COV\left[\widetilde{M}_{t+1},\boldsymbol{\beta}'_{it}\widetilde{\boldsymbol{F}}_{t+1}\right]/E\left[\widetilde{M}_{t+1}\right]. \tag{19}$$

Developing the scalar product of the two vectors indicates that:

$$E\left[1+\tilde{g}_{i,t+1}^{a}\right]=1/E\left[\widetilde{M}_{t+1}\right](1+\rho_i^{-1})$$
$$-COV\left[\widetilde{M}_{t+1},\beta_{1it}\tilde{F}_{1,t+1}+\beta_{2it}\tilde{F}_{2,t+1}+\cdots+\beta_{Nit}\tilde{F}_{N,t+1}\right]/E\left[\widetilde{M}_{t+1}\right]. \tag{20}$$

Again, using simple mathematical covariance properties, the last equation can be arranged and presented as a multilinear function, as we show below:

$$E\left[1+\tilde{g}_{i,t+1}^{a}\right]=\lambda_{0t}/(1+\rho_i^{-1})+\lambda_{1t}\beta_{1it}+\lambda_{2t}\beta_{2it}+\cdots+\lambda_{Nt}\beta_{Nit}, \tag{21}$$

where,

$$\lambda_{0t} \equiv 1/E\big[\widetilde{M}_{t+1}\big],$$

$$\lambda_{kt} \equiv -COV\big[\widetilde{M}_{t+1}, \tilde{F}_{k,t+1}\big]/E\big[\widetilde{M}_{t+1}\big], \text{ for every } k = 1, 2, \ldots, N.$$

Equation (21) now represents an equilibrium condition expressed by several factors, in a single period. This equation is similar in form to the principal prediction of the APT, and like the APT the model tells us nothing about the size or the signs of parameters λ_{kt}. If we consider the aggregate consumption growth rate as a potential factor, then the sign of the corresponding parameter λ_{kt} should be positive because the marginal rate of substitution is negatively related to consumption growth, by construction.[10] Nevertheless, any of the factors can be transformed to produce a positive parameter λ_{kt}.[11]

Earnings growth with many periods and many factors

In the long run (for many periods), the relationship between a firm's abnormal earnings growth rate and its multiple sensitive coefficients can be deduced by summing from time zero ($t = 0$) to time $T - 1$ ($t = T - 1$), as we show below:

$$\sum_{t=0}^{T-1} E\big[1 + \tilde{g}^{a}_{i,t+1}\big] = \sum_{t=0}^{T-1} [\lambda_{0t}/(1 + \rho_i^{-1}) + \lambda_{1t}\beta_{1it} + \lambda_{2t}\beta_{2it} + \cdots + \lambda_{Nt}\beta_{Nit}]. \quad (22)$$

Using the basic properties of the summation operator, we have:

$$\sum_{t=0}^{T-1} E\big[1 + \tilde{g}^{a}_{i,t+1}\big] = \frac{1}{1 + \rho_i^{-1}} \sum_{t=0}^{T-1} \lambda_{0t}$$
$$+ \sum_{t=0}^{T-1} \lambda_{1t}\beta_{1it} + \sum_{t=0}^{T-1} \lambda_{2t}\beta_{2it} + \cdots + \sum_{t=0}^{T-1} \lambda_{Nt}\beta_{Nit}. \quad (23)$$

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[10] See, for example, Bergeron (2013b, p. 192).

[11] See, for example, Campbell (2000, p. 1525), for an equivalent result with returns.

Multiplying by the scalar values $\sum_{t=0}^{N-1} \lambda_{1t}$, $\sum_{t=0}^{T-1} \lambda_{2t}$, …, and $\sum_{t=0}^{T-1} \lambda_{Nt}$, on each side of Equation (23), allows us to write:

$$\sum_{t=0}^{T-1} E\left[1+\tilde{g}_{i,t+1}^{a}\right] = \frac{1}{1+\rho_i^{-1}} \sum_{t=0}^{T-1} \lambda_{0t}$$

$$+\sum_{t=0}^{T-1} \lambda_{1t} \sum_{t=0}^{T-1} w_{1t}\beta_{1it} + \sum_{t=0}^{T-1} \lambda_{2t} \sum_{t=0}^{T-1} w_{2t}\beta_{2it} + \cdots + \sum_{t=0}^{T-1} \lambda_{Nt} \sum_{t=0}^{T-1} w_{Nt}\beta_{Nit}, \tag{24}$$

where $w_{kt} \equiv \lambda_{kt}/\sum_{t=1}^{T-1} \lambda_{kt}$ for every $k = 1, 2, \ldots, N$. Therefore, dividing by the number of the estimated periods T, indicates that:

$$\frac{1}{T}\sum_{t=0}^{T-1} E\left[1+\tilde{g}_{i,t+1}^{a}\right] = \frac{1}{1+\rho_i^{-1}} \frac{1}{T}\sum_{t=0}^{T-1} \lambda_{0t}$$

$$+\frac{1}{T}\sum_{t=0}^{T-1} \lambda_{1t} \sum_{t=0}^{T-1} w_{1t}\beta_{1it} + \frac{1}{T}\sum_{t=0}^{T-1} \lambda_{2t} \sum_{t=0}^{T-1} w_{2t}\beta_{2it} + \cdots + \frac{1}{T}\sum_{t=0}^{T-1} \lambda_{Nt} \sum_{t=0}^{T-1} w_{Nt}\beta_{it}. \tag{25}$$

Using the standard arithmetic average, we can rewrite Equation (25) as follows:

$$1 + \bar{g}_i^a = \lambda_0/(1+\rho_i^{-1}) + \lambda_1\bar{\beta}_{1i} + \lambda_{2t}\bar{\beta}_{2i} + \cdots + \lambda_{Nt}\bar{\beta}_{Ni} \tag{26}$$

where,

$\bar{g}_i^a \equiv \sum_{t=0}^{T-1} E\left[\tilde{g}_{i,t+1}^{a}\right]/T$, $\lambda_0 \equiv \sum_{t=0}^{T-1} \lambda_{0t}/T$,

$\lambda_k \equiv \sum_{t=0}^{T-1} \lambda_{kt}/T$, $\bar{\beta}_{ki} \equiv \sum_{t=0}^{T-1} w_{kt}\beta_{kit}$ (for every $k = 1, 2, \ldots, N$).

In Equation (26), the estimator $\bar{g}_i^a$ represents the long-run arithmetic average of the expected abnormal earnings growth rates of firm *i*. Parameters λ_1 , λ_2 , …, and λ_N, represent, respectively, the arithmetic average of time parameters λ_{1t} , λ_{2t}, …, and λ_{Nt}, over *T* periods. Coefficients $\bar{\beta}_{1i}$, $\bar{\beta}_{2i}$, …, and $\bar{\beta}_{Ni}$, represent, respectively, the weighted average of sensitive coefficients β_{1it}, β_{2it}, and β_{Nit}, over the same number of periods.

To simplify the notation, we use (again) matrix algebra and rewrite Equation (26) in this compact form:

$$1 + \bar{g}_i^a = \lambda_0/(1 + \rho_i^{-1}) + \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}, \tag{27}$$

where $\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}$ is a *column vector* containing the elements $\bar{\beta}_{1i}, \bar{\beta}_{2i}, \dots, \bar{\beta}_{Ni}$, while $\boldsymbol{\lambda}'$ is a *row vector* containing the elements $\lambda_1, \lambda_2, \dots, \lambda_N$.

Equation (27) reveals that the average abnormal earnings growth rate of a firm is linearly and positively related to *N* sensitivity beta coefficients, given by the average sensitivity between earnings and economic factors, over many periods. This result is consistent with the familiar notion that growth is associated with risk, and that fast growing firms (in terms of earnings) tend to be riskier than average.[12] In fact, according to Grullon et al. (2002) and Brav et al. (2005), big old firms that already pay generous dividends present low risk and low expected growth, in the long run.

Intrinsic equity value

More importantly, Equation (27) allows us to calculate the intrinsic equity value of a firm, using many long-run risk factor loadings (or factor sensitivities). Indeed, after simple algebraic manipulations, we can isolate the term ρ_i^{-1} of Equation (27) to obtain:

$$\rho_i^{-1} = \frac{\lambda_0}{1 + \bar{g}_i^a - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}} - 1. \tag{28}$$

Using the same denominator on the right-hand side of equation (28), produces the following:

$$\rho_i^{-1} = \frac{\lambda_0}{1 + \bar{g}_i^a - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}} - \frac{1 + \bar{g}_i^a - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}}{1 + \bar{g}_i^a - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}}. \tag{29}$$

In this manner, we can isolate the expected value ρ_i, as shown below:

[12] See, for example, Beaver et al. (1970).

$$\rho_i = \frac{1 + \bar{g}^a_{i,t+1} - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}}{\lambda_0 + \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}} - \bar{g}^a_i - 1}. \tag{30}$$

At time $t = 0$, Equation (7) establishes that the difference between the market equity value of a firm and its corresponding book value is directly proportional to its current abnormal earnings. This indicates that $\rho_i = (V_{i0} - Y_{i0})/X^a_{i0}$, and that Equation (30) is equivalent to:

$$(V_{i0} - Y_{i0})/X^a_{i0} = \frac{1 + \bar{g}^a_i - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}}{\lambda_0 + \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}} - \bar{g}^a_i - 1}. \tag{31}$$

As a result, the market equity value of a firm can be estimated in this way:

$$V_{i0} = Y_{i0} + \frac{1 + \bar{g}^a_i - \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}}}{\lambda_0 + \boldsymbol{\lambda}'\overline{\boldsymbol{\beta}}_{\boldsymbol{i}} - \bar{g}^a_i - 1} X^a_{i0}, \tag{32}$$

or, if we prefer, in this way:

$$V_{i0} = Y_{i0} + \frac{1 + \bar{g}^a_i - \lambda_1\bar{\beta}_{1i} - \lambda_{2t}\bar{\beta}_{2i} - \cdots - \lambda_{Nt}\bar{\beta}_{Ni}}{\lambda_0 + \lambda_1\bar{\beta}_{1i} + \lambda_{2t}\bar{\beta}_{2i} + \cdots + \lambda_{Nt}\bar{\beta}_{Ni} - \bar{g}^a_i - 1} X^a_{i0}. \tag{33}$$

Equation (33) represents our main result. This equation proposes that the intrinsic value of a firm equals its book value plus an additional amount directly proportional to its current abnormal earnings, positively related to its long-run abnormal earnings growth rate, and negatively related to *N* long-run earnings sensitivity parameters.

Under conditions of certainty, sensitivity parameters equal zero and there is no risk adjustment, in Equation (33) or (32). Under conditions of uncertainty, the sensitive coefficients have a negative effect on the intrinsic equity value, and we can maintain that these coefficients represent a multidimensional risk measure. Thus, the theoretical equity value of a firm appears to be a function of its current book value, abnormal earnings, and *N* risk parameters, given by the long-run sensitivity of earnings to various economic factors.

As in Bergeron et al. (2018) or Bansal et al. (2005), the present model does not assume that dividends or earnings will grow at the same rate in the future. This allows us to integrate a long-run measure of risk similar to Bansal and Yaron (2004), since in the initial long-run risk approach, the cash flow growth rate varies over time, which is consistent with the classical notion that firms have different stages of growth and risk in the long run.[13]

A particular case (*the only factor is aggregate consumption*)

If we suppose that the aggregate consumption growth represents the only factor that generates earnings, then our earnings process, as expressed by Equation (1), exhibits this particular case:

$$\tilde{g}^{a}_{i,t+1} = \alpha_{it} + \beta_{cit}\tilde{g}_{t+1} + \tilde{\varepsilon}_{i,t+1}, \tag{34}$$

where $\tilde{g}_{t+1}$ is the aggregate consumption growth rate between time t and $t+1$, and β_{cit} is the earnings growth rate sensitivity to consumption for firm i at time t. For this unidimensional case, it is easy to prove that parameter β_{cit} is equivalent to:[14]

$$\beta_{cit} = COV\left[\tilde{g}_{t+1}, \tilde{g}^{a}_{i,t+1}\right]/V[\tilde{g}_{t+1}].$$

In this manner, Equation (33) is reduced to the following expression:

$$V_{i0} = Y_{i0} + \frac{1 + \bar{g}^{a}_{i} - \lambda_1\bar{\beta}_{1i}}{\lambda_0 + \lambda_1\bar{\beta}_{1i} - \bar{g}^{a}_{i} - 1}X^{a}_{i0}, \tag{35}$$

where $\tilde{F}_{1,t+1}$ corresponds to $\tilde{g}_{t+1}$ and β_{1it} corresponds to β_{cit}.[15]

Equation (35) suggests that the intrinsic value of a firm equals its book value plus an additional

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[13] See, for example, Mac an Bhaird and Lucey (2011).

[14] If x, y and e represent general variables, and if y = a + bx + e, where COV(x, e) = 0, then COV(x, y) = COV(x, a + bx + e) = COV(x, x)b. Therefore: b = COV(x, y)/σ^2(x).

[15] Notice that if the earnings-consumption covariance is positive (negative), then the corresponding covariance between abnormal earnings and consumption is also positive (negative). This result comes directly from the fact that abnormal earnings are defined by the difference between the random earnings and a constant (p. 6).

amount negatively related to the long-run covariance between earnings and aggregate consumption (reflecting global economic activities). This simple formula is identical to the main result obtained in Bergeron et al. (2018).[16] In this sense, we can assert that the precedent unidimensional-method represents a particular case of the present multidimensional-method. Moreover, the present approach is more robust than the original, for the following reasons: (1) the present model makes no assumption about the joint probability distribution of earnings and consumption; (2) the model employs no strong assumptions about utility function (such as the constant relative risk aversion assumption via the power utility function); (3) the model allows the earnings growth rates to be dependent on many factors, not just one; (4) the model expresses the multidimensionality of risk (with multiple factor sensitivities).

Besides, as mentioned previously, Da (2009) reveals that the long-run covariance between earnings and aggregate consumption represents a key determinant of asset pricing. In this regard, we can also argue that our multifactor approach represents a potential extension of the simple earnings risk setting proposed by Da (2009).

CONCLUSION

Many asset pricing models employ a multifactor approach to characterize risk. In this paper, we extended the residual income valuation model using the long-run sensitivity of earnings to various economics factors. Our procedure integrated the multidimensionality of uncertainty, as well as the long-run concept of risk. First, we assumed that abnormal earnings are generated by several factors. Second, we integrated this generated process into the intertemporal equilibrium version of the residual income valuation framework. Thereafter, we summed over many periods to demonstrate that the abnormal earnings growth rate of a firm is linearly and positively related to N sensitivity coefficients, given by the long-run sensitivity between earnings and economic factors. Finally, we revealed that the intrinsic equity value of a firm is a function of the current book value,

[16] See Bergeron et al. (2018), Equation (49), page 21.

abnormal earnings, and *N* long-run risk parameters. Our development was similar to any multifactor asset pricing model that, given the *N* factors process, deduces asset values from equilibrium conditions. In the context of the residual income valuation approach, these findings suggest that earnings sensitivity to several factors represents an additional technique to estimate risk (in the long run).

The contribution of this paper is essentially theoretical. Future research could use one of our model predictions to develop an empirical test similar to APT tests. For example, it could be useful to test the cross-sectional relation between earnings sensitivity coefficients and earnings growth rates, as predicted by our present model.[17]

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[17] See our Equation (26).

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