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Dividend growth, stock valuation, and long-run risk

Abstract In this paper, we integrate the long-run concept of risk into the stock valuation process. We use the intertemporal consumption capital asset pricing model to demonstrate that a stock's long-run dividend growth is negatively related to its current dividend-price ratio and positively related to its long-run covariance between dividends and consumption. Then, we show that the equilibrium price of a stock is determined by its current dividend, long-run dividend growth, and long-run risk. In all, our work suggests that risk cumulated over many periods represents an important parameter in assessing the theoretical value of a firm.

Keywords Valuation model Dividends Long-run risk Intertemporal model CCAPM.

JEL Classification D91, G12

1 Introduction

According to Bakshi and Chen (2005), literature on stock valuation has not made as much progress as valuation models for derivative and fixed-income securities. Granted that stocks are intrinsically more difficult to value, they claim that asset pricing research has largely focused on expected-return models, but not on stock valuation *per se*. In addition, they claim that an expected-return characterization is insufficient for solving the stock valuation problem.

The purpose of this paper is to develop a stock valuation model based on the intertemporal consumption capital asset pricing model (CCAPM) of Rubinstein (1976), Lucas (1978), and Breeden (1979). As in Bakshi and Chen, the goal of this model development is to derive a stock valuation formula that explicitly relates the stock's fair value to observable fundamental variables. However, our approach differs significantly from Bakshi and Chen, since it values dividends rather than earnings.

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Actually, our model is based on this fundamental result of the CCAPM: the value of a share of stock is equal to the present value of all future stochastic dividends.[2] Valuing dividends, rather than earnings, permits us to avoid two assumptions, used by Bakshi and Chen: (i) dividend equals a fixed fraction of net earnings-per-share plus noise, and (ii) the expected earnings growth rate follows a mean-reverting stochastic process.[3] More important, our dividend approach permits us to integrate the long-run concept of risk, recently proposed in the literature, into the stock valuation process.

Indeed, recent evidence suggests that long-run movements in consumption and dividend constitute a key determinant of the risk-return relationship. For example, Bansal and Yaron (2004) argue that consumption and dividend growth rates contain a small long-run component and depend on the level of economic uncertainty. They show that long-run risk in cash flow should carry higher risk compensation and explain differences in asset expected returns. Bansal et al. (2005) later show that long-run covariance between dividends and consumption (cash flow beta) contains important information regarding the risk premia on assets. In particular, differences in cash flow betas account for more than 60% of the cross-sectional variation in risk premia. Furthermore, when investor’s horizon tends to infinity, Bansal et al. (2009) demonstrate that an asset’s risk is determined almost exclusively by the long-run cointegration between its dividends and consumption.[4]

From the above mentioned studies, we can conclude that the risk of an asset should be estimated over many periods. In this paper, we integrate this long-run definition of risk into the stock valuation process. Rather than focus on expected return, as many studies do, we focus on theoretical value *per se*, as Bakshi and Chen propose.

First, we use the CCAPM framework to demonstrate that a stock’s long-run dividend growth is negatively related to its current dividend-price ratio and positively related to its long-run covariance between dividends and consumption. Then, we show that the equilibrium price of a stock is determined by its current dividend, long-run dividend growth, and long-run risk.

This last result is of particular interest. It indicates that the theoretical value of a stock can be summarized in one new simple formula, which, in addition, is easy to apply. In fact, by knowing the current dividend and the parameters of the utility

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[2] See Rubinstein (1976).
[3] See Bakshi and Chen (2005, p. 111 and p. 112).
[4] See also Parker and Julliard (2005), Hansen et al. (2008), Malloy et al. (2009), Beeler and Campbell (2009), and Bansal and Kiku (2011).

function, the formula simply requires choosing a limited horizon, and, for each period, estimating: 1) the distribution of consumption growth; 2) the stock's expected dividend growth; and 3) the covariance between stock dividend growth and consumption growth, divided by the variance of consumption growth. In so doing, the long-run dividend growth will correspond to the time average expected dividend growth, while the long-run risk will correspond to the time average covariance.

Compared with the classic Gordon (1962) valuation model, our methodology presents important differences. First, the stationary dividend growth rate assumption adopted by Gordon under certainty can be relaxed. Second, our framework explicitly integrates and identifies the measure of risk into the final formula, which comes directly from the manipulation of the first order condition of Rubinstein's (1976) fundamental economic problem.

Another influential stock valuation model developed in the accounting literature is the residual income model of Ohlson (1995). This approach shows that equity value can be split into two components: the accounting current book value and the present value of future discounted cash flows not captured by the current book value (*the residual income*). Nevertheless, this approach differs from ours, since it refers to accounting variables. In this sense, it also differs from the well-known *Earnings multiplier model*, developed by Basu (1977).

The rest of the paper is organized as follows. Section 2 describes the equilibrium framework of our model. Section 3 derives the theoretical value of a stock, via its expected dividend growth. Section 4 concludes the paper.

2 Equilibrium framework

The intertemporal equilibrium framework of our model considers a closed economy populated by identical agents. At time *t*, each agent maximizes the time-separable utility function:

$$E_t \sum_{s=0}^{\infty} \delta^s U(\tilde{C}_{t+s}), \tag{1}$$

subject to resource constraints. Here, δ is the subjective or time discount factor ($0 < \delta < 1$), C_{t+s} ($C_{t+s} > 0$) is consumption at time $t+s$ ($s = 0, 1, 2, ..., \infty$), and

$U(\bullet)$ is an increasing concave and derivable function.[5] In such economy, individual equilibrium allocation must be a solution to the preceding problem, and first order necessary condition can be used to show[6] that the price of asset *i* ($i = 1, 2, ..., N$) at time *t*, P_{it}, is:

$$P_{it} = E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \tilde{D}_{i,t+s}, \quad (2)$$

where $\tilde{D}_{i,t+s}$ represents the dividend of asset *i* at time $t+s$ ($s = 1, 2, ..., \infty$).[7] The right-hand side of equation (2) may be viewed as the fundamental value of a long-lived asset, such as a stock. It reveals that prices equal the present value of all future dividends. In this equation, the stochastic discount factor for each future dividend corresponds to the intertemporal marginal rate of substitution between consumption at time *t* and consumption at time *t*+*s*: $\delta^s U'(\tilde{C}_{t+s}) / U'(C_t)$.

To facilitate the estimation of equation (2), we refer, like Bansal and Kiku (2011), to the standard assumption of a constant relative risk aversion via the power utility function given by:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad (3)$$

where γ (γ > 0) is the coefficient of relative risk aversion (see also, Longstaff and Piazzesi 2004; Kang and Kim 2006; Bayraktar and Young 2007; and many others). With this assumption, the equilibrium price becomes:

$$P_{it} = E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \tilde{D}_{i,t+s}. \quad (4)$$

Since the dividend of stock *i* at time *t*, D_{it}, is known with the current information, it can thus be passed through the conditional expectation operator and be multiplied on each side, to obtain:

$$P_{it} = D_{it} E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \frac{\tilde{D}_{i,t+s}}{D_{it}}, \quad (5)$$

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[5]The operators E_t, VAR_t, and COV_t refer respectively to mathematical expectations, variance, and covariance, where index *t* implies that we consider the available information at time *t* (*t* = 0, 1, 2, …, ∞). The tilde (~) indicates a random variable.

[6] See Rubinstein (1976).

[7] The premium (U') is a derivative of a function.

or

$$P_{it} = D_{it} E_t[\tilde{F}_{it}], \tag{6}$$

where variable $\tilde{F}_{it}$ is defined as follows;

$$\tilde{F}_{it} \equiv \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \frac{\tilde{D}_{i,t+s}}{D_{it}}.$$

To simplify the notation, we can also express equation (6) in this manner:

$$P_{it} = D_{it} \theta_{it}, \tag{7}$$

where $\theta_{it} \equiv E_t[\tilde{F}_{it}]$. Moreover, if the sequence of variables $\tilde{F}_{it}$ (t = 0, 1, 2, …, ∞) is independent and identically distributed (*i.i.d.*), then:

$$P_{it} = D_{it} \theta_i. \tag{8}$$

Finally, given the available information at time *t*, equation (8) allows us to establish that the price of stock *i* at time *t*+1 ($\tilde{P}_{i,t+1}$) and the corresponding dividend are stochastically related, or more precisely that:

$$\tilde{P}_{i,t+1} = \tilde{D}_{i,t+1} \theta_{\mathrm{i}}. \tag{9}$$

3 Theoretical value

Directly from the individual equilibrium allocation problem, we develop, in this section, a simple formula for the valuation of stocks, in which risk is measured in the long-run. In the first part of our development, we concentrate our attention on only one period (between *t* and *t* +1) and later aggregate over many periods. Next, we introduce the riskless asset into the model. Finally, we assume that the dividend growth rate of a stock may be stated as a linear function of the K-periods average of future consumption rates, plus a disturbance term.

One period

It is known that the value of a stock can be expressed for only one period[8]. In fact, recursively, equation (4) shows that:

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[8] See, for example, Huang and Litzenberger (1988, page 202).

$$P_{it} = E_t\left[\delta\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\tilde{D}_{i,t+1}+\tilde{P}_{i,t+1})\right]. \quad (10)$$

Substituting equations (8) and (9) into equation (10) yields:

$$D_{it}\theta_i = E_t\left[\delta\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\tilde{D}_{i,t+1}+\tilde{D}_{i,t+1}\theta_i)\right]. \quad (11)$$

Now, define $\tilde{g}_{i,t+1}$ as the dividend growth rate of stock i between t and t+1, and $\tilde{g}_{t+1}$ as the corresponding consumption growth rate:

$$\tilde{g}_{i,t+1} \equiv \frac{\tilde{D}_{i,t+1}}{D_{it}} - 1, \text{ and } \tilde{g}_{t+1} \equiv \frac{\tilde{C}_{t+1}}{C_t} - 1.$$

Integrating the above growth rates in equation (11) gives, after manipulations:

$$1 = E_t[\delta(1+\tilde{g}_{t+1})^{-\gamma}(1+\tilde{g}_{i,t+1})(1+\theta_i^{-1})]. \quad (12)$$

Taking the expectation on each side, permits us to ignore the full information set at time t, and release the index t of the conditional expectation operator, to write:

$$1 = E[\delta(1+\tilde{g}_{t+1})^{-\gamma}(1+\tilde{g}_{i,t+1})(1+\theta_i^{-1})]. \quad (13)$$

Since $1 = E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]/E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]$, we can also write:

$$0 = E[\delta(1+\tilde{g}_{t+1})^{-\gamma}(1+\tilde{g}_{i,t+1})(1+\theta_i^{-1})] - \frac{E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]}{E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]}. \quad (14)$$

Integrating the last element into the expectation operator and simplifying, gives:

$$0 = E\left[\delta(1+\tilde{g}_{t+1})^{-\gamma}\left\{(1+\tilde{g}_{i,t+1})(1+\theta_i^{-1}) - 1/E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]\right\}\right]. \quad (15)$$

The definition of covariance shows that:

$$COV\left[\delta(1+\tilde{g}_{t+1})^{-\gamma}, (1+\tilde{g}_{i,t+1})(1+\theta_i^{-1}) - 1/E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]\right] =$$

$$-E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]E\left[(1+\tilde{g}_{i,t+1})(1+\theta_i^{-1}) - 1/E[\delta(1+\tilde{g}_{t+1})^{-\gamma}]\right], \quad (16)$$

while the properties of covariance imply that:

$$\delta(1+\theta_i^{-1})COV[(1+\tilde{g}_{t+1})^{-\gamma}, \tilde{g}_{i,t+1}] =$$

$$1-\delta(1+\theta_i^{-1})E[(1+\tilde{g}_{t+1})^{-\gamma}]E[1+\tilde{g}_{i,t+1}]. \tag{17}$$

Thus, in equilibrium, the expected divided growth of any stock satisfied:

$$E[1+\tilde{g}_{i,t+1}] = \frac{1/(1+\theta_i^{-1})}{\delta E[(1+\tilde{g}_{t+1})^{-\gamma}]} - \frac{COV[(1+\tilde{g}_{t+1})^{-\gamma}, \tilde{g}_{i,t+1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]}. \tag{18}$$

To simplify equation (18), we suppose (as in Rubinstein 1976) that the dividend of stock *i* and the aggregate consumption are bivariate normally distributed. Based on the lemma of Stein[9], we can rewrite equation (18) as follows:

$$E[1+\tilde{g}_{i,t+1}] = \frac{1/(1+\theta_i^{-1})}{\delta E[(1+\tilde{g}_{t+1})^{-\gamma}]} + \gamma \frac{E[(1+\tilde{g}_{t+1})^{-\gamma-1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]} COV[\tilde{g}_{t+1}, \tilde{g}_{i,t+1}]. \tag{19}$$

Multiplying both sides of the equation (19) by the variance of the consumption growth rate, $\sigma^2(\tilde{g}_{t+1})$, yields:

$$E[\tilde{g}_{i,t+1}] = -1 + \lambda_{1t}(1+\theta_i^{-1})^{-1} + \lambda_{2t}\beta_{git}, \tag{20}$$

where

$\lambda_{1t} \equiv 1/\delta E[(1+\tilde{g}_{t+1})^{-\gamma}] > 0,$

$\lambda_{2t} \equiv \gamma\sigma^2(\tilde{g}_{t+1})E[(1+\tilde{g}_{t+1})^{-\gamma-1}]/E[(1+\tilde{g}_{t+1})^{-\gamma}] > 0,$

$\beta_{git} \equiv COV(\tilde{g}_{t+1}, \tilde{g}_{i,t+1})/\sigma^2(\tilde{g}_{t+1}).$

The resulting coefficient β_{git} represents, at time *t*, the covariance between the dividend growth rate of stock *i* and the consumption growth rate, divided by the variance of the consumption growth rate. It measures how sensitive a stock's dividend is to aggregate consumption. In this sense, it is very similar to what Bansal et al. (2002, p. 5) call *dividend beta* or what Abel (1999) identify as the key determinant of the risk premia. In other respects, since γ, δ, C_t and $\tilde{C}_{t+1}$ are strictly positive, it follows that λ_{1t} and λ_{2t} are also strictly positive (note that: $1+\tilde{g}_{t+1} = \tilde{C}_{t+1}/C_t$).

[9] If x and y are bivariate normally distributed: $COV(y, f(x)) = E(f'(x))COV(y, x)$. See Huang and Litzenberger (p. 101).

Many periods

Starting the economy at time zero and summing from t = 0 to t = T-1, gives:

$$\sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] = \sum_{t=0}^{T-1} (-1 + \lambda_{1t}(1+\theta_i^{-1})^{-1} + \lambda_{2t}\beta_{git}) \tag{21}$$

or

$$\sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] = -T + (1+\theta_i^{-1})^{-1} \sum_{t=0}^{T-1} \lambda_{1t} + \sum_{t=0}^{T-1} \lambda_{2t}\beta_{git} \,. \tag{22}$$

Multiplying by $\sum_{t=0}^{T-1} \lambda_{2t}$ on each side, yields:

$$\sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] = -T + (1+\theta_i^{-1})^{-1} \sum_{t=0}^{T-1} \lambda_{1t} + \sum_{t=0}^{T-1} \lambda_{2t} \sum_{t=0}^{T-1} w_t \beta_{git} \,, \tag{23}$$

where $w_t \equiv \lambda_{2t} / \sum_{t=0}^{T-1} \lambda_{2t}$, with $\sum_{t=0}^{T-1} w_t = 1$. At time t = 0, equation (8) shows that: $\theta_i^{-1} = D_{i0} / P_{i0}$; where the left hand side of the equality represents the current dividend-price ratio. Thus, multiplying by T^{-1} on each side of equation (23) shows that:

$$\bar{g}_i = -1 + \lambda_1 (1 + D_{i0} / P_{i0})^{-1} + \lambda_2 \bar{\beta}_{gi} \,, \tag{24}$$

where

$$\bar{g}_i \equiv \sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] / T \,,$$

$$\bar{\beta}_{gi} \equiv \sum_{t=0}^{T-1} w_t \beta_{git} \,,$$

$$\lambda_1 \equiv \sum_{t=0}^{T-1} \lambda_{1t} / T \,, \quad \lambda_2 \equiv \sum_{t=0}^{T-1} \lambda_{2t} / T \,, \quad \lambda_1 > 0, \quad \lambda_2 > 0.$$

Here, $\bar{g}_i$ can be viewed as the arithmetic average (over many periods) of the expected dividend growth rates of stock i, or, to put it differently, the long-run expected dividend growth rate of stock i. In the same way, $\bar{\beta}_{gi}$ can be view as the weighted average of the coefficients β_{git} (t = 0, 1, 2, …, T-1), or, more simply, the *long-run beta* of stock i.

In this manner, equation (24) shows that the long-run expected dividend growth rate of a stock is a linear function of its current dividend-price ratio and its long-run beta (or its long-run covariance between dividends and consumption). In addition, since λ_{1t} and λ_{2t} are strictly positive for any *t* (*t* = 0, 1, 2, …, *T*-1) then λ_1 and λ_2 are necessarily positive. Thus, a stock's long-run expected dividend growth appears to be negatively related to its current dividend-price ratio and positively related to its long-run beta.

Rearranging equation (24) finally relates the current price, P_{i0}, of the stock with its current dividend, D_{i0}, that is to say:

$$P_{i0} = \frac{1+\bar{g}_i - \lambda_2 \bar{\beta}_{gi}}{\lambda_1 + \lambda_2 \bar{\beta}_{gi} - \bar{g}_i - 1} D_{i0} . \tag{25}$$

More precisely, equation (25) shows that P_{i0} is a function of D_{i0}, $\bar{g}_i$, and $\bar{\beta}_{gi}$. Since D_{i0} and $\bar{g}_i$ have a positive effect on P_{i0}, while $\bar{\beta}_{gi}$ has a negative effect, we can argue that the last parameter represents a risk adjustment factor and a stock's measure of risk. Consequently, the equilibrium price of a stock appears to be a function of its current dividend, long-run expected dividend growth, and long-run risk, as measured by its long-run beta.

In the same way, the long-run expected dividend growth appears to be negatively related to its current dividend-price ratio and positively related to its long-run risk.

The link between dividend growth, current dividend, and risk (in its standard expression) is not a new subject in finance. For example, in Beaver, Kettler and Scholes (1970, p. 661), we can read:

> Firms with lower payout ratios, *ceteris paribus*, will have higher growth rates. Yet it was argued above that low payout implies greater riskiness. If so, then growth rate would be positively associated with risk.

Similarly, in Senbet and Thompson (1982, p. 332) we can see[10]:

> Brigham and Gordon [3], Gordon [8], and others have found very strong inverse correlation between dividend yield and expected dividend growth. Brennan and Sharpe and Sosin [16] have found high inverse correlation between dividend yield and beta. Putting these two pieces of evidence together, we have positive correlation between growth and beta or risk.

[10] Rozeff (1982), Eades (1982), Baskin (1989), Gillet et al. (2008), Carter and Schmidt (2008), and many others, present similar results regarding the dividend-risk relationship.

Our result, summed up by equation 24, is consistent with these above findings.

It is also consistent with the following argument: If we postulate that expected returns as expected capital gains are positively related to risk, and if we assume that prices and dividends are cointegrated[11], then expected dividend growth and risk must be positively related. The belief can, furthermore, be rationalized in the following manner: If firms are risk averse and prudent, then those operating in a high level of uncertainty will be reluctant to pay high current dividend and will prefer to redistribute earnings later. Thus, in a high level of uncertainty, firms will display simultaneously a high measure of risk, a low current dividend (relative to earnings, price or future dividends), and a high expected dividend growth.

In short, our result is consistent with the notion that big old firms that already pay generous dividends and have low risk (Brav et al. 2005; Grullon et al., 2002) present low expected dividend growth in the long-run.

Also, from the formula, expressed by equation 25, the determination of a stock's theoretical value implies the following steps: 1) choice of a horizon (T); 2) observation of the current dividend D_{i0}; 3) establishment of parameters λ_1 and λ_2 from the utility function and the distribution of the aggregate dividend growth; and 4) estimation of the long-run expected dividend growth rate $\overline{g}_i$, and the long-run beta $\overline{\beta}_{gi}$. To estimate the long-run expected dividend growth rate, investors will have to make a prediction for each period and then, compute the average. A simple way to do this is to assume, for example, that an extraordinary growth will continue for a certain number of years, after which growth will change to an ordinary level. In the same manner, the estimation of the long-run beta covers many periods.

Therfore, our asset valuation model integrates the long-run concept of risk initially proposed by Bansal et al. (2002), Bansal and Yaron (2004), Bansal et al. (2005), Parker and Julliard (2005), Bansal (2007), Bansal et al. (2009), and others.

Riskless asset

We can facilitate our estimation of parameters λ_{1t} and λ_{2t}, if we assume the existence of a riskless asset. In fact, from equation (10) it follows that:

$$1 = E\left[\delta\left(\frac{\tilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right](1 + r_{f,t+1}), \tag{26}$$

[11] See, for example, Esteve and Prats (2010), for a more complete definition of cointegration or our equations (8) and (9).

where $r_{f,t+1}$ represents the risk-free rate of return, between t and t+1. As a result, the definition of λ_{1t} shows that:

$$\lambda_{1t} = 1 + r_{f,t+1} . \tag{27}$$

In addition, for market portfolio m, equations (8) and (20) show that:[12]

$$E[\tilde{g}_{m,t+1}] = -1 + \lambda_{1t}(1 + d_{mt})^{-1} + \lambda_{2t}\beta_{gmt} , \tag{28}$$

where d_{mt} equals D_{mt} / P_{mt} and represents the dividend-price ratio (or the dividend yield) of the portfolio market, at time t. Rearranging, gives:

$$\lambda_{2t} = [1 + E[\tilde{g}_{m,t+1}] - (1 + r_{f,t+1})(1 + d_{mt})^{-1}] / \beta_{gmt} . \tag{29}$$

Thus, the estimation of parameters λ_{1t} and λ_{2t} can provide from the observations of the riskless asset and the market portfolio.

Linear function

We can also facilitate our estimation of long-run risk if we make an additional assumption. Indeed, let the variable $\tilde{x}_t$ be the K-periods average of future consumption, or, more precisely, let:

$$\tilde{x}_t = (1/K)\sum_{k=1}^{K} \tilde{g}_{t+k} , \tag{30}$$

where, $\tilde{g}_{t+k} = (\tilde{C}_{t+k} / \tilde{C}_{t+k-1}) - 1$. Now, assume that the relationship between dividend and consumption growth rates is given by the following linear function:

$$\tilde{g}_{i,t+1} = a_i + b_i\tilde{x}_t + \tilde{\varepsilon}_{i,t+1} , \tag{31}$$

in which $E[\tilde{\varepsilon}_{i,t+1}] = 0,$ and $E[\tilde{g}_{t+k}\tilde{\varepsilon}_{i,t+1}] = 0$. With this restrictive assumption, it is easy to demonstrate (see the Appendix A) that:

$$P_{i0} = \frac{1 + \bar{g}_i - \lambda_2' b_i}{\lambda_1 + \lambda_2' b_i - \bar{g}_i - 1} D_{i0} . \tag{32}$$

[12] In this paper, the indice m replace i, when refering to market portfolio.

where $\lambda'_2 \equiv \sum_{t=0}^{T-1} \lambda'_{2t} / T$, and $\lambda'_{2t} \equiv COV[\tilde{g}_{t+1}, \tilde{x}_t]\gamma E[(1+\tilde{g}_{t+1})^{-\gamma-1}]/E[(1+\tilde{g}_{t+1})^{-\gamma}]$.

Parameter b_i can be interpreted as a standard regression coefficient that measures the covariance between dividend growth and long future consumption growth. It is equal to:

$$b_i = \frac{COV[\tilde{x}_t, \tilde{g}_{i,t+1}]}{\sigma^2[\tilde{x}_t]}. \quad (33)$$

Moreover, this parameter can be viewed as a measure of long-run covariance between dividend growth and consumption growth, since:

$$COV[\tilde{x}_t, \tilde{g}_{i,t+1}] = (1/K)\sum_{k=1}^{K} COV[\tilde{g}_{t+k}, \tilde{g}_{i,t+1}]. \quad (34)$$

Therefore, for this particular case, we can argue that our definition of risk is relatively close to what Bansal et al. (2002 and 2005) call *cash flow beta*.[13]

Otherwise, if we accept the existence of a riskless asset, then parameter λ'_{2t} could be estimated as follows (see Appendix B):

$$\lambda'_{2t} = [1 + E[\tilde{g}_{m,t+1}] - (1 + r_{f,t+1})(1 + d_{mt})^{-1}] / b_m, \quad (35)$$

where b_m equals:

$$b_m = \frac{COV[\tilde{x}_t, \tilde{g}_{m,t+1}]}{\sigma^2[\tilde{x}_t]}.$$

Finally, following Bansal and Yaron (2004), we can argue that economic uncertainty or fluctuation represents a reasonable reason for why consumption growth rates are not *i.i.d.*, and why: (1) the expectation of growth in aggregate consumption would differ from one year to the next; (2) the parameters λ_{1t} or λ_{2t} and the weights, in the weighted average of the betas, change over time; and (3) the betas in the future would be different from what is observed today.

[13] See, in particular, Bansal et al. (2002, p. 5 and p. 6).

4 Conclusion

Our primary goal was to integrate the long-run concept of risk into the stock valuation process. Using the intertemporal framework of the CCAPM, we have demonstrated that a stock's long-run dividend growth is negatively related to its current dividend-price ratio and positively related to its long-run beta, calculated by the long-run covariance between dividends and consumption. Then, we have shown that the theoretical value of a stock is a function of its current dividend, long-run expected dividend growth, and long-run beta. Finally, we have argued that long-run beta should represent a measure of risk, since it exerts a negative effect on price. These results come directly from the individual equilibrium allocation problem, and do not need stationary process for the dividend or the consumption growth rate. Overall, our model supports the view that risk cumulated over many periods influences the intrinsic value of a firm and its equity.

In this paper, we refer to the standard assumption of a constant relative risk aversion via the power utility function. It could be interesting, for future research, to relax this assumption. One appealing generalization of power utility could be, for example, the Epstein and Zin's (1989) preferences, or the habit formation of Sundaresan (1989) and Constantinides (1990), or Abel's (1990 and 1999) «catching up with the Joneses». Moreover, it could be interesting to prove that our model does not require the assumption of a normal distribution.

Appendix A

In this appendix, we assume that the dividend growth rate on a stock may be stated as a linear function of the K-periods average of future consumption rates, plus a disturbance term.

From equation (21) we have:

$$\sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] = \sum_{t=0}^{T-1}\left(-1 + \frac{\lambda_{1t}}{(1+\theta_i^{-1})} + \gamma \frac{E[(1+\tilde{g}_{t+1})^{-\gamma-1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]} COV[\tilde{g}_{t+1}, \tilde{g}_{i,t+1}]\right). \quad \text{(A1)}$$

Introducing equation (31) in (A1) gives:

$$\sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] =$$

$$\sum_{t=0}^{T-1}(-1+\frac{\lambda_{1t}}{(1+\theta_i^{-1})}+\gamma\frac{E[(1+\tilde{g}_{t+1})^{-\gamma-1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]}COV[\tilde{g}_{t+1},a_i+b_i\tilde{x}_t+\tilde{\varepsilon}_{i,t+1}]). \qquad \text{(A2)}$$

The properties of the covariance show that:

$$\sum_{t=0}^{T-1}E[\tilde{g}_{i,t+1}]=\sum_{t=0}^{T-1}(-1+\frac{\lambda_{1t}}{(1+\theta_i^{-1})}+\gamma\frac{E[(1+\tilde{g}_{t+1})^{-\gamma-1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]}COV[\tilde{g}_{t+1},\tilde{x}_t]b_i) \qquad \text{(A3)}$$

or

$$\sum_{t=0}^{T-1}E[\tilde{g}_{i,t+1}]=-T+(1+\theta_i^{-1})^{-1}\sum_{t=0}^{T-1}\lambda_{1t}+b_i\sum_{t=0}^{T-1}\lambda'_{2t}. \qquad \text{(A4)}$$

where $\lambda'_{2t} \equiv COV[\tilde{g}_{t+1},\tilde{x}_t]\gamma E[(1+\tilde{g}_{t+1})^{-\gamma-1}]/E[(1+\tilde{g}_{t+1})^{-\gamma}]$. Multiplying by T^{-1} in each side of equation (A4) yields:

$$\bar{g}_i = -1+\lambda_1(1+D_{i0}/P_{i0})^{-1}+\lambda'_2 b_i \qquad \text{(A5)}$$

where $\lambda'_2 \equiv \sum_{t=0}^{T-1}\lambda'_{2t}/T$. Thus:

$$P_{i0}=\frac{1+\bar{g}_i-\lambda'_2 b_i}{\lambda_1+\lambda'_2 b_i-\bar{g}_i-1}D_{i0}. \qquad \text{(A6)}$$

Appendix B

We can also facilitate our estimation of parameters λ'_{2t}, if we assume the existence of a riskless asset. In fact, for the market portfolio, equation (31) shows that:

$$\tilde{g}_{m,t+1}=a_m+b_m\tilde{x}_t+\tilde{\varepsilon}_{m,t+1}, \qquad \text{(B1)}$$

in which $E[\tilde{\varepsilon}_{m,t+1}]=0$, $E[\tilde{g}_{t+k}\tilde{\varepsilon}_{m,t+1}]=0$, and:

$$b_m=\frac{COV[\tilde{x}_t,\tilde{g}_{m,t+1}]}{\sigma^2[x_t]}.$$

For the market portfolio, equation (8) and (20) also indicate that:

$$E[1+\tilde{g}_{m,t+1}]=\frac{\lambda_{1t}}{(1+d_{mt})}-\gamma\frac{E[(1+\tilde{g}_{t+1})^{-\gamma-1}]}{E[(1+\tilde{g}_{t+1})^{-\gamma}]}COV[(\tilde{g}_{t+1},\tilde{g}_{m,t+1}]. \qquad \text{(B2)}$$

Introducing equation (B1) in (B2) gives, after simplifications:

$$E[\tilde{g}_{m,t+1}] = -1 + \lambda_{1t}(1 + d_{mt})^{-1} + \lambda'_{2t} b_m . \tag{B3}$$

Thus:

$$\lambda'_{2t} = [1 + E[\tilde{g}_{m,t+1}] - (1 + r_{f,t+1})(1 + d_{mt})^{-1}] / b_m . \tag{B4}$$

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