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Inflation, Risk, and Equilibrium Asset Returns

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Abstract

In this note, we examine the theoretical effect of inflation and risk on asset returns. From the fundamental prediction of the capital asset pricing model, we first relate the expected nominal rate of return of the asset to its inflation beta (estimated by the covariance between the asset's nominal rate of return and the rate of inflation, divided by the variance of the inflation rate). Then, we show that the equilibrium expected rate of return on risky asset can be expressed by a linear combination of its standard beta and inflation beta. This result indicates that inflation rate, in addition to market return, influences asset returns. This result also suggests that inflation risk, in addition to market risk, should be priced in the cross-section of asset returns.

Keywords: Inflation, Risk, Inflation Beta, Capital Asset Pricing Model, Inflation-CAPM, Multifactor Model

1. Introduction

The theoretical effect of inflation and risk on asset returns represents a fairly significant issue that has generated the intense interest of various researchers. First, Chen and Boness (1975) point out that the standard capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) is derived without an explicit consideration of inflation. Assuming a mean-variance framework similar to Mossin (1966) and a specific preference structure for investors, Chen and Boness derive a simple formula that characterizes the equilibrium relationship between the risk and the expected return of stocks, under the condition of uncertain inflation. Simalarly, Friend et al. (1976) derive the effects of uncertain inflation on the equilibrium demand relation for risky assets, using a continuous time model. Elton et al. (1983) then break out of the limitations of the mean variance assumptions by using the Arbitrage Pricing Theory (APT) of Ross (1976). They show that if the process generating real returns is affected by both the return on the market and the rate of inflation, a model for equilibrium real returns can be derived.

Empirically, many evidences on the link between inflation, risk and asset returns have been observed. For instance, in their empirical test of the APT, Chen et al. (1986) find that several macroeconomic variables, including inflation, affect stock returns. In particular, they find that inflation is priced by the market (see also, Priestley,1996; Campbell and Vuolteenaho, 2004; and Shanken and Weinstein, 2006). Moreover, some studies, such as those of Wong and Wu (2003) and Chang (2013), support the Fisher's classic hypothesis concerning the positive relationship between stock returns and inflation; however, other studies, such as those of Fama (1977), Fama and Gibbons (1982) and Andrangi and Chatrath (2002), suggest that the correlation is negative.

More recently, some economic explanations have been developed to explore the link between inflation and returns. For example, Gabaix (2008) suggests that inflation is priced in the market because when disaster occurs, inflation tends to increase. Bansal and Shaliastovich (2013) propose that inflation influences asset returns because inflation is exposed to the same real shocks that drive consumption and long-run risk. Duarte (2013) argues that investors are willing to accept lower unconditional returns when holding securities that are good hedge against inflation. Actually, he finds that stocks whose returns covary negatively with inflation shocks have unconditional higher returns (see also Katzur and Spierdijk, 2013).

The purpose of this note is to develop a theoretical model on the relationship between inflation, risk and expected asset returns.

Our development presents important differences compared with the theoretical models mentioned above. First, our construction starts from the main prediction of the CAPM. Second, our framework does not refer to any specific utility function or return generating process. More particularly, we do not: (1) use the restrictive quadratic utility function (Chen and Boness, 1975); (2) assume that the random rate of inflation is generated by a continuous Gaussian-Wiener process (Friend et al., 1976); (3) make an assumption on the return generating process (Elton et al., 1983); or (4) presume that consumption follows a restrictive auto regressive process (Duarte, 2013)..

The derivation of our model can be summarized as follows. We postulate, in accordance with the zero-beta CAPM of Black (1972), that the expected equilibrium nominal rate of return on a given risky asset, is equal to the return required in the marketplace for portfolios that have no systematic risk, plus a risk premium directly proportional to its standard beta. Next, we note that the nominal interest rate can be expressed as the sum of a real return and an inflation rate. Then, using the basic properties of covariance, we formulate a special version of the Euler equation and demonstrate that the equilibrium expected nominal rate of return of a risky asset is linearly related to its inflation beta (estimated by the covariance between the asset's nominal rate of return and the rate of inflation, divided by the variance of the inflation rate).

In this manner, we show that the equilibrium expected rate of return on a risky asset can be expressed by a linear combination of its standard beta and inflation beta. This result indicates that inflation rate, in addition to market return, influences asset returns. This result also suggests that inflation risk, in addition to market risk, should be priced in the cross-section of stock returns.

The rest of this paper is spilt into four sections. The next section relates the expected nominal rate of return of a risky asset to its inflation beta. The third section shows the link between expected returns, inflation betas and standard betas. The fourth section briefly discusses the possibility of developing a multifactor extension of the model. The last section concludes the paper.

2. CAPM and Inflation

The classic CAPM starts from the assumption that investors are generally risk averse and shows that, in equilibrium, capital assets will be priced such that[1]:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i\,, \tag{1}$$

where $\tilde{R}_i$ is the rate of return of asset i, $\tilde{R}_z$ is the rate of return of the zero beta-portfolio, $\tilde{R}_m$ is the rate of return of the market portfolio, and β_i is the standard beta of asset i[2]. Since $\beta_i \equiv COV[\tilde{R}_m, \tilde{R}_i]/\sigma^2[\tilde{R}_m]$, we can also rewrite equation (1) as follows[3]:

[1] See Black (1972).
[2] The tilde (~) indicates a random variable. Operator E refers to mathematical expectations.
[3] Operators σ^2 and COV refer respectively to variance and covariance.

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])COV[\tilde{R}_m, \tilde{R}_i]/\sigma^2[\tilde{R}_m]. \tag{2}$$

By definition, if $\tilde{\pi}$ represents the inflation rate and $\tilde{R}_m^r$ represents the real rate of return of the market portfolio, then the corresponding nominal rate for the market portfolio is such that: $1+\tilde{R}_m =$ $(1+\tilde{R}_m^r)(1+\tilde{\pi})$. Therefore:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])COV[\tilde{\pi}\tilde{R}_m^r + \tilde{R}_m^r + \tilde{\pi}, \tilde{R}_i]/\sigma^2[\tilde{R}_m]. \tag{3}$$

This allows us to write that:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\{COV[\tilde{\pi}\tilde{R}_m^r + \tilde{R}_m^r, \tilde{R}_i] + COV[\tilde{\pi}, \tilde{R}_i]\}/\sigma^2[\tilde{R}_m]. \tag{4}$$

This also allows us to isolate the coefficient of the covariance between the asset's nominal rate of return and the rate of inflation. That is:

$$COV[\tilde{\pi}, \tilde{R}_i] = \sigma^2[\tilde{R}_m](E[\tilde{R}_i] - E[\tilde{R}_z])/(E[\tilde{R}_m] - E[\tilde{R}_z]) - COV[\tilde{\pi}\tilde{R}_m^r + \tilde{R}_m^r, \tilde{R}_i]. \tag{5}$$

From the general definition of covariance, we obtain:

$$COV[\tilde{\pi}, \tilde{R}_i] = E[\tilde{\pi}\tilde{R}_i] - E[\tilde{\pi}]E[\tilde{R}_i] = \frac{\sigma^2[\tilde{R}_m](E[\tilde{R}_i] - E[\tilde{R}_z])}{E[\tilde{R}_m] - E[\tilde{R}_z]} - COV[\tilde{\pi}\tilde{R}_m^r + \tilde{R}_m^r, \tilde{R}_i]. \tag{6}$$

Now, if we define $\tilde{R}_u$ as the return on a portfolio that is uncorrelated with inflation[4], or, if we prefer, a portfolio such that the covariance between inflation and portfolio's return equals zero, then:

$$COV[\tilde{\pi}, \tilde{R}_u] = E[\tilde{\pi}\tilde{R}_u] - E[\tilde{\pi}]E[\tilde{R}_u] = 0. \tag{7}$$

Equation (6) minus Equation (7) shows that:

$$COV[\tilde{\pi}, \tilde{R}_i - \tilde{R}_u] = E[\tilde{\pi}(\tilde{R}_i - \tilde{R}_u)] - E[\tilde{\pi}](E[\tilde{R}_i] - E[\tilde{R}_u]) \tag{8}$$

or:

$$E[\tilde{\pi}(\tilde{R}_i - \tilde{R}_u)] = COV[\tilde{\pi}, \tilde{R}_i - \tilde{R}_u] + E[\tilde{\pi}](E[\tilde{R}_i] - E[\tilde{R}_u]). \tag{9}$$

Consequently, Equation (9) gives, after simple manipulations, a particular form of the Euler equation[5] in which the central random variables are explicitly expressed by the inflation rate, the nominal rate of return of the asset, and the nominal rate of return of the portfolio that is uncorrelated with inflation. That is to say:

$$E[\tilde{\pi}\tilde{X}_i] = 1 \tag{10}$$

where $\tilde{X}_i \equiv \dfrac{\tilde{R}_i - \tilde{R}_u}{COV[\tilde{\pi}, \tilde{R}_i] + E[\tilde{\pi}](E[\tilde{R}_i] - E[\tilde{R}_u])}$.

Note that this particular unconditional form of the Euler equation can easily be expressed with a conditional expected value, if we believe that return distribution varies over time, and if we believe that the CAPM (Equation 1) holds conditionally on the information set available at time *t*[6]. At the aggregate level, for the market portfolio, *m*, we can also write that:

[4] Elton et al. (1983) defines a similar portfolio on page 528.
[5] For an example of the (consumption) Euler equation, see Equation (1) in Mulligan (2004).
[6] See, for example, Lewellen and Nagel (2006), for a discussion on Conditional-CAPM validity.

$$E[\tilde{\pi}\tilde{X}_m] = 1, \tag{11}$$

where $\tilde{X}_m \equiv \dfrac{\tilde{R}_m - \tilde{R}_u}{COV[\tilde{\pi}, \tilde{R}_m] + E[\tilde{\pi}](E[\tilde{R}_m] - E[\tilde{R}_u])}$.

Equation (10) minus Equation (11) yields:

$$E[\tilde{\pi}(\tilde{X}_i - \tilde{X}_m)] = 0. \tag{12}$$

From the definition of covariance, we obtain:

$$COV[\tilde{\pi}, \tilde{X}_i - \tilde{X}_m] = -E[\tilde{\pi}]E[\tilde{X}_i - \tilde{X}_m] = 0. \tag{13}$$

Also, rearranging the last equation shows that:

$$E[\tilde{X}_i] = E[\tilde{X}_m] + COV[\tilde{\pi}, \tilde{X}_m]/E[\tilde{\pi}] - COV[\tilde{\pi}, \tilde{X}_i]/E[\tilde{\pi}]. \tag{14}$$

In accordance with the previous form of the Euler equation, we can rewrite Equation (14) in this manner:

$$\begin{aligned} E[\tilde{R}_i] - E[\tilde{R}_u] &= E[\tilde{X}_m]\{COV[\tilde{\pi}, \tilde{R}_i] + E[\tilde{\pi}](E[\tilde{R}_i] - E[\tilde{R}_u])\} \\ &+ \{COV[\tilde{\pi}, \tilde{R}_i] + E[\tilde{\pi}](E[\tilde{R}_i] - E[\tilde{R}_u])\}COV[\tilde{\pi}, \tilde{X}_m]/E[\tilde{\pi}] - COV[\tilde{\pi}, \tilde{X}_i]/E[\tilde{\pi}]. \end{aligned} \tag{15}$$

Hence, after simple manipulations, we get:

$$\begin{aligned} E[\tilde{R}_i] - E[\tilde{R}_u] &= (E[\tilde{R}_i] - E[\tilde{R}_u])(E[\tilde{X}_m]E[\tilde{\pi}] + COV[\tilde{\pi}, \tilde{X}_m]) \\ &+ COV[\tilde{\pi}, \tilde{R}_i](E[\tilde{X}_m] + COV[\tilde{\pi}, \tilde{X}_m]/E[\tilde{\pi}] - 1/E[\tilde{\pi}]) \end{aligned} \tag{16}$$

or:

$$E[\tilde{R}_i] - E[\tilde{R}_u] = \frac{E[\tilde{X}_m] + COV[\tilde{\pi}, \tilde{X}_m] - 1/E[\tilde{\pi}]}{1 - E[\tilde{X}_m]E[\tilde{\pi}] + COV[\tilde{\pi}, \tilde{X}_m]} COV[\tilde{\pi}, \tilde{R}_i]). \tag{17}$$

Equation (17) shows that the excess return of asset *i*, on portfolio *u*, is directly proportional to its covariance between inflation and return. Likewise, multiplying both sides of Equation (17) by the variance of the inflation rate, $\sigma^2[\tilde{\pi}]$, gives:

$$E[\tilde{R}_i] - E[\tilde{R}_u] = \frac{(E[\tilde{X}_m] + COV[\tilde{\pi}, \tilde{X}_m] - 1/E[\tilde{\pi}])\sigma^2[\tilde{\pi}]}{1 - E[\tilde{X}_m]E[\tilde{\pi}] + COV[\tilde{\pi}, \tilde{X}_m]} \; \frac{COV[\tilde{\pi}, \tilde{R}_i])}{\sigma^2[\tilde{\pi}]}. \tag{18}$$

Equation (18) can thus be arranged and presented as a simple linear relationship. That is to say:

$$E[\tilde{R}_i] = E[\tilde{R}_u] + \lambda_\pi \beta_i^\pi, \tag{19}$$

with

$$\lambda_\pi \equiv \frac{(E[\tilde{X}_m] + COV[\tilde{\pi}, \tilde{X}_m] - 1/E[\tilde{\pi}])\sigma^2[\tilde{\pi}]}{1 - E[\tilde{X}_m]E[\tilde{\pi}] + COV[\tilde{\pi}, \tilde{X}_m]},$$

$$\beta_i^\pi \equiv COV[\tilde{\pi}, \tilde{R}_i]/\sigma^2[\tilde{\pi}].$$

Parameter β_i^π is especially interesting. It is called the inflation beta, which is equal to the covariance between the asset's nominal rate of return and the rate of inflation, divided by the variance of the inflation rate. It is viewed as the portion of risk associated to inflation. It measures the asset's return sensitivity to inflation. A positive value of β_i^{π} indicates that asset *i* is likely to have a higher return when inflation exists (and vice versa).

Beside, as noted by Duarte and Mishara-Blomberger (2012), the definition of inflation betas implies that they are the coefficient of a linear univariate regression of realized returns on inflation. As a result, an inflation beta equal to 1 means that the nominal return on asset *i* varies in a one-to-one correspondence with the inflation rate.

So, Equation (19) shows that the equilibrium expected nominal rate of return on a risky asset equals the nominal return for a portfolio that is uncorrelated with inflation, plus a risk premium. Here, the risk premium consists of two elements. The first, β_i^{π}, represents the quantity of inflation risk for asset *i*. The second, λ_{π}, is the same for all assets and serves as a weighting factor for the asset's inflation risk factor and it can be viewed as *the market price of inflation risk*. For the market portfolio, *m*, we can also write:

$$E[\tilde{R}_m] = E[\tilde{R}_u] + \lambda_{\pi}\beta_m^{\pi}, \tag{20}$$

with:

$$\beta_m^{\pi} \equiv COV[\tilde{\pi}, \tilde{R}_m]/\sigma^2[\tilde{\pi}].$$

where β_m^{π} is the inflation beta of the market portfolio. Therefore:

$$\lambda_{\pi} = (E[\tilde{R}_m] - E[\tilde{R}_u]) / \beta_m^{\pi}, \tag{21}$$

and:

$$E[\tilde{R}_i] = E[\tilde{R}_u] + (E[\tilde{R}_m] - E[\tilde{R}_u])\beta_i^{\pi} / \beta_m^{\pi}. \tag{22}$$

Equation (22) is similar to the standard CAPM equation, except that instead of using the returns of the market portfolio as the only risk factor, it uses inflation rate as Duarte (2013) or Duarte and Mishara-Blomberger (2012) propose[7]. In its form, it is also very similar to the risk-return relationship expressed by the Consumption-based CAPM of Bredeen (1979)[8].

From Equation (22), the inflation risk of an asset now appears to be measured by its inflation beta divided (or normalized) by the inflation beta of the market portfolio. Thus, in average, it is equal to 1, just like the standard beta, which facilitates the interpretation.

Also, if we believe, as many empirical studies show, that the stock's inflation beta or the market inflation beta is usually negative[9], then the normalized inflation beta, $\beta_i^{\pi} / \beta_m^{\pi}$, must have a positive value. Moreover, in Equation (22), the market price of risk appears to be equivalent to the spread between the expected market return and the expected return of the portfolio that has no systematic inflation risk, which also facilitates the interpretation.

3. Expected returns, inflation betas and market betas

Equation (22) comes directly from the main prediction of the standard CAPM, expressed by Equation (1). This allows us to write that:

$$2E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i + E[\tilde{R}_u] + (E[\tilde{R}_m] - E[\tilde{R}_u])\beta_i^{\pi} / \beta_m^{\pi}. \tag{23}$$

[7] See, in particular, Equation (1) of Duarte and Mishara-Blomberger (2012).
[8] See Equation (19) on page 276, in Bredeen (1979).
[9] See, for example, Chen et al. (1986).

Rearranging, we have:

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{2}\beta_{\mathrm{i}} + \frac{E[\tilde{R}_m] - E[\tilde{R}_u]}{2}\beta_i^{\pi} / \beta_m^{\pi}, \tag{24}$$

or, if we prefer:

$$E[\tilde{R}_i] = \lambda_0 + \lambda_1\beta_i + \lambda_2\beta_i^{\pi}, \tag{25}$$

with

$$\lambda_0 = (E[\tilde{R}_z] + E[\tilde{R}_u])/2$$
$$\lambda_1 = (E[\tilde{R}_m] - E[\tilde{R}_z])/2$$
$$\lambda_2 = (E[\tilde{R}_m] - E[\tilde{R}_u])/2\beta_m^{\pi}.$$

Equation (25) shows that the equilibrium expected rate of return on a risky asset can be expressed by a linear combination of its standard beta and inflation beta.

This result appears to be fully consistent with the APT, in the case where the return-generating process is a function of two economic factors: market returns and inflation rates. However, our result does not specifically come from this restrictive return-generating process. It comes directly from the main prediction of the CAPM and from the basic definition of the real rate of return. Actually, contrary to the APT, our derivation does not need to assume that: (1) the returns on any risky asset is linearly related to a set of *N* factors, (2) the number of this factors is two, and (3) these two factors are those given above. In other words, our result does not need to suppose a specific return-generating process.

Coefficients β_i and β_i^{π}, in Equation (25), can be viewed as the sensitivity of the returns on the *i*th security to the market factor and inflation factor, respectively. They can also be viewed as risk quantities (market systematic risk and inflation systematic risk). Thus, a natural interpretation for λ_1, and λ_2 is that they represent the premium (the price of risk), in equilibrium, for the market factor and inflation factor, respectively.

Risk premium λ_1 is equal to the difference between the expectation of the market portfolio returns and the expectation of the zero-beta portfolio, divided by two. Its magnitude is then two times smaller than the risk premium given by the standard CAPM. This is explained by the fact that the market risk represents only a part of the risk (the other part is given by inflation). In practice, its estimation is as simple as it is for the standard CAPM.

Risk premium λ_2 is equal to the difference between the expectation of the market portfolio returns and the expectation of the portfolio that is uncorrelated with inflation, divided by two, and weighted by the inflation beta of the market portfolio. As before, its magnitude takes into account the fact that the risk of each asset depends on two factors. In addition, its magnitude is directly proportional to the sensitivity of the market portfolio returns to the inflation factor. If there is a positive difference between the expectation of the market portfolio returns and the expectation of the portfolio that is uncorrelated with inflation, and if there is a positive covariance between the rate of return on the market and the rate of inflation, then the inflation risk premium appears to be positive. (The reverse is true with negative values.)

Besides, if we prefer characterizing the risk-return relationship with equation (24), then the quantity of inflation risk will appear to be equal to β_i / β_i^{π} (the normalized inflation beta) and *the market price of inflation risk* will appear to be equal to $(E[\tilde{R}_m] - E[\tilde{R}_u])/2$.

As for intercept, λ_0, its value simply corresponds to the average between the expected rate of return of the zero-beta portfolio and the expectation of the portfolio that is uncorrelated with inflation. It also corresponds to the expected rate of return of an asset that presents no systematic risk, or if we prefer, such that: $\beta_i = \beta_i^{\pi} = 0$.

Equation (24) also shows that if the random rate of return of an asset is equal to the rate of return of the market portfolio ($\tilde{R}_i = \tilde{R}_m$), then its market beta and normalized inflation beta are equivalent to one ($\beta_i = \beta_i / \beta_i^{\pi} = 1$) and its equilibrium rate is such that: $E[\tilde{R}_i] = E[\tilde{R}_m]$.

Moreover, if the covariance between the asset's nominal rate of return and the rate of inflation is equal to zero, for every asset, then the corresponding inflation beta also equals zero. Therefore, Equation (19) shows that:

$$E[\tilde{R}_i] = E[\tilde{R}_u], \tag{26}$$

while Equation (25) indicates that:

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{(E[\tilde{R}_m] - E[\tilde{R}_z])}{2}\beta_{\mathrm{i}}\,. \tag{27}$$

Integrating Equation (26) into Equation (27), gives

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z]}{2} + \frac{E[\tilde{R}_i]}{2} + \frac{(E[\tilde{R}_m] - E[\tilde{R}_z])}{2}\beta_{\mathrm{i}} = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i\,. \tag{28}$$

Consequently, the standard CAPM, expressed by Equation (1), appears to be a particular case of our model. It is the case when inflation and asset returns are uncorrelated (for every asset) or when inflation is assumed to be known with certainty. It is also the case when inflation and market portfolio returns are perfectly correlated (see Appendix A).

In brief, in the more realistic case where inflation rate is uncertain, where individual asset returns covariate significantly with inflation and where market portfolio returns are not perfectly correlated with inflation, then the equilibrium expected rate of return of a risky asset is equivalent to Equation (25). This result, thus, represents an additional simple formula that can be helpful in characterizing the equilibrium relationship between the risk and the expected returns of assets, under the condition of uncertain inflation.

4. Extension model

In this section, we discuss the possibility of developing a multifactor extension of our model. Our presentation refers to Campbell (2000, p. 1525-1526).

To extend to a multifactor model, we first assume that the return on market portfolio is generated by a set of industry indices. More precisely, we assume, for expositional simplicity, that the rate of return on the market portfolio is a linear function of *K* indices as shown below:

$$\tilde{R}_m = b_0 + b_1 \tilde{I}_1 + b_2 \tilde{I}_2 + \ldots + b_K \tilde{I}_K\,, \tag{29}$$

where $\tilde{I}_k$ is the index k that influences market returns, b_0 is the intercept associated to the market multifactor model, and b_k is the sensitivity of the market returns to the index k (k = 1,2, …, K).

In the same manner, we assume that the rate of inflation is generated by a set of economic factors. More particularly, we assume that the rate of inflation is a linear function of *N* factors as shown in equation (30):

$$\tilde{\pi} = b_0^{\pi} + b_1^{\pi}\tilde{F}_1 + b_2^{\pi}\tilde{F}_2 + ... + b_N^{\pi}\tilde{F}_N \,, \tag{30}$$

where $\tilde{F}_n$ is the factor *n* that influences inflation rates, b_0^{π} is the intercept associated to the inflation multifactor model, b_n^{π} is the sensitivity of the inflation rates to the factor *n* (n = 1, 2, …, N).

If we integrate equation (29) and equation (40) in equation (25), then it's easy to demonstrate (see appendix B) that the equilibrium expected rate of return of asset *i* can be written in this way:

$$E[\tilde{R}_i] = \lambda_0 + \sum_{k=1}^{K} \lambda_k^I \beta_{ki} + \sum_{n=1}^{N} \lambda_n^{\pi} \beta_{ni}^{\pi} \,, \tag{31}$$

where:

$$\beta_{ki} \equiv COV[\tilde{I}_k \,, \tilde{R}_i]/\sigma^2[\tilde{I}_k\,],\; \lambda_k^I \equiv \lambda_1 b_k \sigma^2[\tilde{I}_k\,]/\sigma^2[\tilde{R}_m]\,,$$
$$\beta_{ni}^{\pi} \equiv COV[\tilde{F}_n \,, \tilde{R}_i]/\sigma^2[\tilde{F}_n\,],\; \lambda_n^{\pi} \equiv \lambda_1 b_n^{\pi} \sigma^2[\tilde{F}_n\,]/\sigma^2[\tilde{\pi}].$$

Here β_{ki} is the coefficient beta of asset return *i* on the industry index *k*, λ_k^I is the price of the risk of the index *k*, β_{ni}^{π} is the coefficient beta of asset return *i* on the economic factor *n*, and λ_k^I is the price of the risk of the factor *n*.

As a result, equation (31) indicates that the equilibrium expected rate of return on a risky asset can be expressed by a linear combination of *K* betas, associated to the *K* industrial indices that influence market returns, plus *N* betas, associated to the *N* economic factors that determine the inflation rate.

5. Conclusion

There are many empirical studies on the relationship between inflation betas and asset returns, in the literature. Our primary contribution with this short note was to examine the theoretical effect of inflation and risk on asset returns, in the context of the CAPM. Starting from the main prediction of the CAPM, we demonstrated that the required rate of return of a risky asset is equal to the expected rate of return of a portfolio that is uncorrelated with inflation, plus a risk premium proportional to the asset's inflation beta. Since our result comes directly from the CAPM, this allowed us to show that the equilibrium expected rate of return on risky asset can be expressed by a linear combination of its standard beta and inflation beta. Overall, our simple extension of the CAPM supports the view that inflation rate, in addition to market return, influences asset returns. It also supports the view that inflation risk, in addition to market risk, should be priced in the cross-section of asset returns.

In the last section of this paper, we have also pointed out the possibility of an extension model that could use more than two factors (or indices). However, our demonstration was based on a very restrictive return generating process. It could be interesting, for future research, to generalize this process, and identify this factors.

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Appendix A

In this appendix A, we show that the standard CAPM, expressed by Equation (1), also appears to be a particular case of our model when inflation and market portfolio returns are perfectly correlated. Indeed, in this case the corresponding absolute correlation coefficient equals one ($CORR[\tilde{\pi}, \tilde{R}_m] = |1| =$ +1 or -1) and the relationship between the two variables is perfectly linear. This allows us to write that:

$$\tilde{\pi} = a + b\tilde{R}_m , \tag{A.1}$$

where a and b represent the coefficients of a general linear function ($b \neq 0$). Integrating this relationship into Equation (24), we can now write that:

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{2}\beta_{\mathrm{i}} + \frac{E[\tilde{R}_m] - E[\tilde{R}_u]}{2}\frac{COV[a + b\tilde{R}_m, \tilde{R}_i]/\sigma^2[\tilde{\pi}]}{COV[a + b\tilde{R}_m, \tilde{R}_m]/\sigma^2[\tilde{\pi}]} . \tag{A.2}$$

Using proprieties of covariance, gives:

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{2}\beta_{\mathrm{i}} + \frac{E[\tilde{R}_m] - E[\tilde{R}_u]}{2}\frac{COV[\tilde{R}_m, \tilde{R}_i]b}{COV[\tilde{R}_m, \tilde{R}_m]b} . \tag{A.3}$$

Therefore:

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{2}\beta_{\mathrm{i}} + \frac{E[\tilde{R}_m] - E[\tilde{R}_u]}{2}\beta_{\mathrm{i}} , \tag{A.4}$$

$$E[\tilde{R}_i] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} + \frac{2E[\tilde{R}_m] - E[\tilde{R}_z] - E[\tilde{R}_u]}{2}\beta_{\mathrm{i}} . \tag{A.5}$$

In this particular case, the risk-return relationship can thus be characterized with only one risk factor (here the standard beta). In addition, for the zero-beta portfolio, the last equation shows that:

$$E[\tilde{R}_z] = \frac{E[\tilde{R}_z] + E[\tilde{R}_u]}{2} . \tag{A.6}$$

This implies that: $E[\tilde{R}_z] = E[\tilde{R}_u]$. This also implies that:

$$E[\tilde{R}_i] = \frac{2E[\tilde{R}_z]}{2} + \frac{2(E[\tilde{R}_m] - E[\tilde{R}_z])}{2}\beta_{\mathrm{i}} = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i , \tag{A.7}$$

which represents, again, the standard CAPM, with the zero-beta portfolio.

Appendix B

In this appendix B, we present the derivation of equation (31). Indeed, integrating equations (29) and (30) into equation (24), shows that:

$$E[\tilde{R}_i] = \lambda_0 + \lambda_1 COV[b_0 + b_1\tilde{I}_1 + b_2\tilde{I}_2 + \ldots + b_K\tilde{I}_K, \tilde{R}_i]/\sigma^2[\tilde{R}_m] +$$
$$\lambda_2 COV[b_0^\pi + b_1^\pi\tilde{F}_1 + b_2^\pi\tilde{F}_2 + \ldots + b_N^\pi\tilde{F}_N, \tilde{R}_i]/\sigma^2[\tilde{\pi}], \tag{B.1}$$

or:

$$E[\tilde{R}_i] = \lambda_0 + \frac{\lambda_1}{\sigma^2[\tilde{R}_m]}\sum_{k=1}^{K} b_k COV[\tilde{I}_k, \tilde{R}_i] + \frac{\lambda_2}{\sigma^2[\tilde{\pi}]}\sum_{n=1}^{N} b_n^\pi COV[\tilde{F}_n, \tilde{R}_i], \tag{B.2}$$

or if we prefer:

$$E[\tilde{R}_i] = \lambda_0 + \sum_{k=1}^{K} \lambda_k^I COV[\tilde{I}_k, \tilde{R}_i]/\sigma^2[\tilde{I}_k] + \sum_{n=1}^{N} \lambda_n^\pi COV[\tilde{F}_n, \tilde{R}_i]/\sigma^2[\tilde{F}_n], \tag{B.3}$$

with,

$$\lambda_k^I \equiv \lambda_1 b_k \sigma^2[\tilde{I}_k]/\sigma^2[\tilde{R}_m],$$
$$\lambda_n^\pi \equiv \lambda_1 b_n^\pi \sigma^2[\tilde{F}_n]/\sigma^2[\tilde{\pi}].$$

Thus, it is easy to recognize equation (31) if we define β_{ki} and β_{ni}^π in this manner:

$$\beta_{ki} \equiv COV[\tilde{I}_k, \tilde{R}_i]/\sigma^2[\tilde{I}_k],$$
$$\beta_{ni}^\pi \equiv COV[\tilde{F}_n, \tilde{R}_i]/\sigma^2[\tilde{F}_n].$$