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Equilibrium Dividend Growth and Stock Valuation

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Abstract

In this paper, we develop a stock valuation model that takes into account the equilibrium dividend growth rate. Our development is based on the capital asset pricing model. We begin by showing that under the condition of equilibrium the expected dividend growth rate of a stock is linearly and positively related to the covariance between stock's dividends and market dividends. We then suggest that the price of a stock is a function of its current dividend, required return, and dividend growth rate, given by the equilibrium condition. In short, our model offers an additional tool to estimate the expected dividend growth of a stock and the corresponding intrinsic value.

Keywords: Dividend, Stock Valuation, Risk, Capital Asset Pricing Model (CAPM), Equilibrium Conditions

1. Introduction

The classic Gordon (1962) valuation model suggests that the intrinsic value of a stock is determined by its required return and dividend growth rate. To help estimate the required returns, an investor or financial analyst can count on several solid equilibrium models, such as the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), the Arbitrage Pricing Theory (APT) of Ross (1976) or the Consumption-CAPM of Breeden (1979). They can also count on an innumerous quantity of empirical research on the relationship between risk and returns.[1] However, when compared to the above-mentioned studies, there is practically no research on estimating expected dividend growth.

In this paper, we develop a stock valuation model that takes into account the equilibrium dividend growth rate.

The development of our model can be summarized as follows. First, we postulate, in accordance with the zero-beta CAPM of Black (1972), that the expected equilibrium rate of return on a given risky stock, is equal to the return required in the marketplace for portfolios that have no systematic risk, plus a risk premium that is directly proportional to its standard beta. Afterward, using basic proprieties of mathematical covariance, we separate dividends from capital gain in the definition of returns, and isolate the covariance between stock's dividends and market dividends. Then, we apply a set of simple algebraic manipulations to isolate the expected dividend growth rate of the stock.

In this manner, we show that under equilibrium the expected dividend growth rate of a stock is linearly and positively related to the covariance between stock's dividends and market dividends. More

[1] For a discussion on the empirical relationship between risk and returns, see, for example, Campbell (2000) or Cochrane (2011).

precisely, the equilibrium dividend growth rate of the stock appears to be equal to the expected dividend growth rate for the zero-beta portfolio, plus a premium. This premium is given by the difference between the expected dividend growth rates of the market and the zero-beta portfolio, multiplied by the sensitivity of stock's dividends to market dividends. This relation thus implies that stock dividend growth should be superior (inferior) to the average when its sensitivity to the market is higher (lower) than the average.

Next, assuming that the expected dividend growth rate of a stock is equivalent to its capital gain rate, which is, in fact, a direct consequence of the Gordon model, we suggest that the stock price is a function of its current dividend, required return, and dividend growth rate as expressed by the equilibrium condition.

In short, our model offers an additional tool to estimate the expected dividend growth of a stock and the corresponding intrinsic value.

As mentioned by Elton et al. (2014), the correct way to estimate the intrinsic value of common stocks has occupied a huge amount of effort over a long period of time.[2] Indeed, following the classic Gordon model, many other models have been proposed to estimate the value of a stock. For example, Brooks and Helms (1990) consider a multistage model with dividend growth rates changing deterministically among the stages. Moreover, Hurley and Johnson (1994, 1998) extend the dividend discount model, assuming that dividends follow a Markov process.[3] Additionally, the well-known residual-income method, popularized by Ohlson (1995), suggests that future cash flows can be estimated using the clean surplus relation. This procedure indicates that equity value can be split into two components: the accounting current book value and the present value of future discounted cash flows not captured by the current book value (*the residual income*).

Donaldson and Kamstra (1996) also extend the Gordon model, using statistical models of discounted dividend growth rates. Further, Feltham and Ohlson (1999) provide a general version of the residual income method in introducing risk and stochastic interest rates. In addition, Pastor and Veronesi (2003) derive a simple approach to valuing stocks in the presence of learning about average profitability. Furthermore, Bakshi and Chen (2005) present a stock valuation model in which the expected earnings growth rate follows a mean-reveting process. Dong and Hirshleifer (2005) then generalize the work of Bakshi and Chen in proposing a stock valuation model that is not restricted to positive-earnings companies.

From another point of view, Yee (2008, 2010) suggests a Bayesian framework for combining two or more estimates into a superior valuation estimate.

More recently, Bergeron (2013-a) develops a valuation model that integrates the long-run definition of consumption risk into the stock valuation process, and Bergeron (2013-b) extends this model in integrating the long-run sensitivity of dividends to various economic factors.

However, none of the above-mentioned works proposed a stock valuation model that explicitly derives the equilibrium dividend growth rate directly from the risk-return relationship described by the CAPM.

Besides, compared to Bergeron (2013-a) and Bergeron (2013-b), the present model shows significant differences. First, our framework is simpler than the intertemporal framework used in the two preceding papers. Second, our model doesn't require a complete determination of the correct utility function or a complete estimation of the aggregate consumption, which facilitates its application.

The rest of this paper is spilt into four sections. The next section derives the equilibrium dividend growth rate of a stock. The third section presents the stock valuation model. The fourth section exhibits a practical application. The last section concludes the paper.

[2] See chapter 18 of Elton et al. (2014).
[3] See, also, Guglielmo (2012).

2. Equilibrium Dividend Growth

The zero-beta CAPM of Black (1972) starts from the assumption that investors are generally risk averse and shows that capital assets will be priced, in equilibrium, as such:[4]

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i\,, \tag{1}$$

where

$\tilde{R}_i$ = the rate of return of asset *i*

$\tilde{R}_z$ = the rate of return of the zero-beta portfolio

$\tilde{R}_m$ = the market rate of return

β_i = the standard beta of asset *i* ($\beta_i \equiv COV[\tilde{R}_m, \tilde{R}_i]/\sigma^2[\tilde{R}_m]$).

Introducing the mathematical definition of the standard beta into Equation (1), allows us to rewrite the main prediction of the CAPM as follows:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])COV[\tilde{R}_m, \tilde{R}_i]/\sigma^2[\tilde{R}_m]\,. \tag{2}$$

If P_{0i} is the price of stock *i* at time 0, $\tilde{P}_{1i}$ is the price of stock *i* at time 1, and $\tilde{D}_{1i}$, is the dividend of stock *i* at time 1, then the rate of return can be decomposed, and Equation (2) can be presented in this way:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])(\sigma^2[\tilde{R}_m])^{-1}COV[\tilde{G}_m + \tilde{d}_m, \tilde{G}_i + \tilde{d}_i]\,, \tag{3}$$

where

$\tilde{G}_i$ = the capital gain of stock *i* ($\tilde{G}_i \equiv [\tilde{P}_{1i} - P_{0i}]/P_{0i}$)

$\tilde{d}_i$ = the dividend yield of stock *i* ($\tilde{d}_i \equiv \tilde{D}_{1i}/P_{0i}$)

$\tilde{G}_m$ = the market capital gain

$\tilde{d}_m$ = the market dividend yield.

Using the basic proprieties of mathematical covariance, we can also decompose the last term of Equation (3) to show:

$$\begin{aligned}&(E[\tilde{R}_i] - E[\tilde{R}_z])\sigma^2[\tilde{R}_m]/(E[\tilde{R}_m] - E[\tilde{R}_z])\\ &\quad = COV[\tilde{G}_m, \tilde{G}_i] + COV[\tilde{G}_m, \tilde{d}_i] + COV[\tilde{d}_m, \tilde{G}_i] + COV[\tilde{d}_m, \tilde{d}_i]\,.\end{aligned} \tag{4}$$

Likewise, if D_{0m} represents the market dividend at time 0, P_{0m} represents the market value, of all stocks, at time 0, and D_{0i} represents the dividend of stock *i* at time 0, then we have:

$$\begin{aligned}&(E[\tilde{R}_i] - E[\tilde{R}_z])\sigma^2[\tilde{R}_m]/(E[\tilde{R}_m] - E[\tilde{R}_z]) - COV[\tilde{G}_m, \tilde{R}_i] - COV[\tilde{d}_m, \tilde{G}_i]\\ &\quad = COV[(1+\tilde{g}_m)D_{0m}/P_{0m}, (1+\tilde{g}_i)D_{0i}/P_{0i}]\,,\end{aligned} \tag{5}$$

where

[4] In our development, the tilde (~) indicates a random variable. Also, the operators *E*, σ^2 and *COV* refer, respectively, to mathematical expectations, variance and covariance.

$\tilde{g}_i$ = the dividend growth rate of stock *i* ($\tilde{g}_i \equiv [\tilde{D}_{1i} - D_{0i}]/D_{0i}$)
$\tilde{g}_m$ = the market dividend growth rate.

Therefore, the covariance between stock's dividend growth rate and market dividend growth rate can be isolated, as shown below:

$$\left[\frac{(E[\tilde{R}_i]-E[\tilde{R}_z])\sigma^2[\tilde{R}_m]}{E[\tilde{R}_m]-E[\tilde{R}_z]} - COV[\tilde{G}_m,\tilde{R}_i] - COV[\tilde{d}_m,\tilde{G}_i]\right]\frac{P_{0m}}{D_{0m}}\frac{P_{0i}}{D_{0i}}$$
$$= COV[\tilde{g}_m,\tilde{g}_i]. \tag{6}$$

Nothing that μ_z represents the expected dividend growth rate of the zero-beta portfolio, and, thus, represents a constant, then the basic proprieties of mathematical covariance allows us to write that:

$$\left[\frac{(E[\tilde{R}_i]-E[\tilde{R}_z])\sigma^2[\tilde{R}_m]}{E[\tilde{R}_m]-E[\tilde{R}_z]} - COV[\tilde{G}_m,\tilde{R}_i] - COV[\tilde{d}_m,\tilde{G}_i]\right]\frac{P_{0m}}{D_{0m}}\frac{P_{0i}}{D_{0i}}$$
$$= COV[\tilde{g}_m,\tilde{g}_i - \mu_z + \mu_z]$$
$$= COV[\tilde{g}_m,\tilde{g}_i - \mu_z]. \tag{7}$$

Taken directly from the basic mathematical definition of covariance, we can see that:

$$\left[\frac{(E[\tilde{R}_i]-E[\tilde{R}_z])\sigma^2[\tilde{R}_m]}{E[\tilde{R}_m]-E[\tilde{R}_z]} - COV[\tilde{G}_m,\tilde{R}_i] - COV[\tilde{d}_m,\tilde{G}_i]\right]\frac{P_{0m}}{D_{0m}}\frac{P_{0i}}{D_{0i}}$$
$$= E[\tilde{g}_m(\tilde{g}_i - \mu_z)] - E[\tilde{g}_m]E[\tilde{g}_i - \mu_z]. \tag{8}$$

Equation (8) minus equation (7) gives:

$$0 = E[\tilde{g}_m(\tilde{g}_i - \mu_z)] - E[\tilde{g}_m]E[\tilde{g}_i - \mu_z] - COV[\tilde{g}_m,\tilde{g}_i - \mu_z]. \tag{9}$$

Consequently, Equation (10) shows, after simple manipulations, a familiar compact equation in which the central random variables are explicitly expressed by the market dividend growth rate, the dividend growth rate of the stock, and the dividend growth rate of the portfolio that is uncorrelated with the market. That is to say:

$$E[\tilde{g}_m\tilde{Z}_i] = 1, \tag{10}$$

where $\tilde{Z}_i \equiv \dfrac{\tilde{g}_i - \mu_z}{COV[\tilde{g}_m,\tilde{g}_i] + E[\tilde{g}_m](E[\tilde{g}_i] - \mu_z)}$.

In the same manner, on an aggregate level, for the market portfolio, *m*, we can write that:

$$E[\tilde{g}_m\tilde{Z}_m] = 1, \tag{11}$$

where $\tilde{Z}_m \equiv \dfrac{\tilde{g}_m - \mu_z}{\sigma^2[\tilde{g}_m] + (E[\tilde{g}_m])^2 - E[\tilde{g}_m]\mu_z}$.

Equation (10) minus Equation (11), yields:

$$E[\tilde{g}_m(\tilde{Z}_i - \tilde{Z}_m)] = 0\,. \tag{12}$$

From the definition of covariance, we obtain:

$$E[\tilde{Z}_i - \tilde{Z}_m] = -COV[\tilde{g}_m, \tilde{Z}_i - \tilde{Z}_m]/E[\tilde{g}_m]\,. \tag{13}$$

Rearranging, the last equation indicates that:

$$E[\tilde{Z}_i] = E[\tilde{Z}_m] + COV[\tilde{g}_m, \tilde{Z}_m]/E[\tilde{g}_m] - COV[\tilde{g}_m, \tilde{Z}_i]/E[\tilde{g}_m]\,. \tag{14}$$

In accordance with the expression and the components of the compact Equation (10) we can rewrite the last relationship in the following manner:

$$\begin{aligned} E[\tilde{g}_i] - E[\tilde{g}_z] \\ &= E[\tilde{Z}_m](COV[\tilde{g}_m, \tilde{g}_i] + E[\tilde{g}_m](E[\tilde{g}_i] - E[\tilde{g}_z])) \\ &+ COV[\tilde{g}_m, \tilde{Z}_m](COV[\tilde{g}_m, \tilde{g}_i] + E[\tilde{g}_m](E[\tilde{g}_i] - E[\tilde{g}_z]))/E[\tilde{g}_m] \\ &- COV[\tilde{g}_m, \tilde{g}_i]/E[\tilde{g}_m]\,. \end{aligned} \tag{15}$$

Hence, after simple algebraic manipulations, we get:

$$\begin{aligned} E[\tilde{g}_i] - E[\tilde{g}_z] \\ &= E[\tilde{Z}_m]E[\tilde{g}_m](E[\tilde{g}_i] - E[\tilde{g}_z]) \\ &+ E[\tilde{Z}_m]COV[\tilde{g}_m, \tilde{g}_i] \\ &+ COV[\tilde{g}_m, \tilde{Z}_m]E[\tilde{g}_i] \\ &+ COV[\tilde{g}_m, \tilde{Z}_m]COV[\tilde{g}_m, \tilde{g}_i]/E[\tilde{g}_m] \\ &- COV[\tilde{g}_m, \tilde{g}_i]/E[\tilde{g}_m] \\ &- COV[\tilde{g}_m, \tilde{Z}_m]E[\tilde{g}_z], \end{aligned} \tag{16}$$

or:

$$\begin{aligned} E[\tilde{g}_i] - E[\tilde{g}_z] \\ &= (E[\tilde{g}_i] - E[\tilde{g}_z])(E[\tilde{Z}_m]\,E[\tilde{g}_m] + COV[\tilde{g}_m, \tilde{Z}_m]) \\ &+ COV[\tilde{g}_m, \tilde{g}_i](E[\tilde{Z}_m] + COV[\tilde{g}_m, \tilde{Z}_m]/E[\tilde{g}_m] - 1/\,E[\tilde{g}_m])\,, \end{aligned} \tag{17}$$

or if we prefer:

$$E[\tilde{g}_i] - E[\tilde{g}_z] = \frac{E[\tilde{Z}_m] + (COV[\tilde{g}_m, \tilde{Z}_m] - 1)/E[\tilde{g}_m]}{1 - E[\tilde{Z}_m]E[\tilde{g}_m] - COV[\tilde{g}_m, \tilde{Z}_m]} COV[\tilde{g}_m, \tilde{g}_i]\,. \tag{18}$$

In the same way, for the market portfolio, we have:

$$E[\tilde{g}_m] - E[\tilde{g}_z] = \frac{E[\tilde{Z}_m] + (COV[\tilde{g}_m, \tilde{Z}_m] - 1)/E[\tilde{g}_m]}{1 - E[\tilde{Z}_m]E[\tilde{g}_m] - COV[\tilde{g}_m, \tilde{Z}_m]} COV[\tilde{g}_m, \tilde{g}_m]\,. \tag{19}$$

Introducing Equation (19) into Equation (18) and rearranging shows that:

$$E[\tilde{g}_i] - E[\tilde{g}_z] = (E[\tilde{g}_m] - E[\tilde{g}_z])COV[\tilde{g}_m, \tilde{g}_i])/\sigma^2[\tilde{g}_m]\,. \tag{20}$$

Finally, we get:

$$E[\tilde{g}_i] = E[\tilde{g}_z] + (E[\tilde{g}_m] - E[\tilde{g}_z])\beta_{gi}, \tag{21}$$

where $\beta_{gi} \equiv COV[\tilde{g}_m, \tilde{g}_i]/\sigma^2[\tilde{g}_m]$.

Equation (21) represents our first significant result. It shows that it is possible to estimate the expected dividend growth rate of a stock using a simple equation. In this form, the equation given above is very similar to the well-known prediction of the CAPM. Basically, it indicates that under equilibrium the expected dividend growth rate of a stock is linearly related to the covariance between stock's dividends and market dividends.

More specifically, at equilibrium, the expected dividend growth rate on any stock is equal to the expected dividend growth rate for the zero-beta portfolio, plus a premium. This premium is given by the difference between the expected market dividend growth rate and the expected dividend growth rate of the zero-beta portfolio, multiplied by parameter β_{gi}.

The last parameter is especially interesting. We call it *dividend growth beta*. It is the covariance between the dividend growth rate on the stock and the market dividend growth rate, divided by the variance of the market dividend growth rate. For the market portfolio (the average), the value is one because its covariance with itself is identical to its variance. To put it differently, the dividend growth beta measures the asset's dividend sensitivity to market dividends. A positive value of the parameter indicates that stock *i* is likely to have a higher dividend growth rates when dividend growth rates on the market is higher (and vice versa).

The dividend growth beta is also similar to the concept of *long-run risk* or *cash flow beta*, recently proposed in the literature.[5] Indeed, for Bansal et al. (2005) cash flow beta measures the long-run covariance between cash flow growth and aggregate consumption growth, where cash flows represent dividends (see page 1644). Moreover, for the fundamental Consumption-CAPM, aggregate consumption equals aggregate dividends (or market dividends). In this sense, the dividend growth beta appears to be very similar to the coefficient known as *dividend beta* in Bergeron (2013-a, page 552) which measures how sensitive a stock's dividend is to aggregate consumption (see also Bergeron 2013-b). Nevertheless, these beta parameters were derived in an intertemporal framework that is relatively more complex than the one period framework employed here.

This begs the following question. What is the direction of the relation between equilibrium dividend growth rate and dividend growth beta? In others words, what is the derivative of *expected dividend growth rate* with respect to *dividend growth beta*? This is an important question because the model shows, a priori, no indication on the spread between the expected dividend growth rate of the market portfolio and the corresponding rate for the zero-beta portfolio. However, the answer is clear: the relationship is positive, since the sign of the derivative is superior to zero. Indeed, from the fundamental prediction of the CAPM, expressed by equation (1), it is easy to prove (see appendix A) that:

$$\frac{\partial E[\tilde{g}_i]}{\partial \beta_{gi}} = \sigma^2[\tilde{g}_m](D_{0m}/P_{0m})\frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{\sigma^2[\tilde{R}_m]}.$$

[5] According to Beeler and Campbell (2012), the long-run concept of risk has attracted a great deal of attention, since the important work of Bansal and Yaron (2004) and Bansal et al. (2005).

Since the expected return of the market portfolio is superior to the expected return of the zero-beta portfolio, and since all the others elements of the preceding derivative equation are positive, by construction, then the derivative is also positive. That is to say:

$$\frac{\partial E[\tilde{g}_i]}{\partial \beta_{gi}} > 0.$$

From equation (21) we can see that if the dividend growth beta of a stock is zero, then its corresponding expected dividend growth should be equal to the expected dividend growth rate of the zero-beta portfolio. Additionally, if the dividend growth beta of a stock is one, or if its dividends are perfectly correlated with market dividends, then its expected dividend growth rate should be equal to the dividend growth rate of the market portfolio which represents the average growth rate for the entire market. In short, stock dividend growth should be superior (inferior) to the average when its sensitivity to the market is higher (lower) than the average.

A priori, the estimated *dividend growth beta* is not more difficult or complicated than the estimated *standard beta*. Actually, the current theoretical estimate for the standard beta should be more complicated, because it requires knowing the hazardous distribution of next prices, in addition to the distribution of next dividends. Dividend growth beta estimation needs only to focus on next dividends, as we see in the equilibrium relationship described by equation (21).

Up to this point, it is important to notice that we have made no specific assumptions in our model development except for the implicit assumption that stock prices and dividends are positive. In fact, we have only postulated that the zero-beta CAPM of Black (1972), expressed by equation (1), is valid. We then simply used: (1) the two components of returns (capital gain and dividends); (2) covariance proprieties; (3) algebraic manipulations, and (4) basic rules of differentiation.

In the next section, we will apply our first results for the valuation of common stocks.

3. Stock Valuation Model

We know that the expected returns can be decomposed between capital gain and dividends. Therefore, from equation (1), we can write that:

$$E[\tilde{R}_i] = E[\tilde{G}_i] + E[\tilde{D}_{1i}]/P_{0i} = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i\,. \tag{22}$$

Respecting the single time period framework of the CAPM, we can use the preceding relationship to determine what the current value of the stock should be. Indeed, by rearranging the above expression, we get:

$$P_{0i} = \frac{E[\tilde{D}_{1i}]}{E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i - E[\tilde{G}_i]}\,. \tag{23}$$

Here, P_{0i}, can be interpreted as the equilibrium price of the risky stock. To facilitate its estimation, we simply suppose that the expected growth rate in stock price equals the expected dividend growth rate of the stock. This allows us to show that:

$$P_{0i} = \frac{E[\tilde{D}_{1i}]}{E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i - E[\tilde{g}_i]}\,. \tag{24}$$

Introducing equation (21) into equation (24) shows that:

$$P_{0i} = \frac{E[\tilde{D}_{1i}]}{E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i - E[\tilde{g}_z] - (E[\tilde{g}_m] - E[\tilde{g}_z])\beta_{gi}}, \tag{25}$$

or preferably:

$$P_{0i} = \frac{D_{0i}(1 + E[\tilde{g}_i])}{E[\tilde{R}_i] - E[\tilde{g}_i]} \tag{26}$$

with

$$E[\tilde{R}_i] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_i$$

$$E[\tilde{g}_i] = E[\tilde{g}_z] + (E[\tilde{g}_m] - E[\tilde{g}_z])\beta_{gi} .$$

Equation (26) summarizes our stock valuation process. It indicates that the equilibrium price of a stock depends on its current dividend, equilibrium expected return, and equilibrium expected dividend growth rate. Its form is very similar to the classic Gordon model when the expected return comes from the CAPM. The innovation here is in the integration of the equilibrium expected dividend growth rate which is also given by the CAPM.

Because the parameters $E[\tilde{R}_z]$, $E[\tilde{R}_m]$, $E[\tilde{g}_z]$ and $E[\tilde{g}_m]$ could be viewed as exogenous variables, the equilibrium price of a stock can be expressed by the following function:

$$P_{0i} = f(D_{0i}, E[\tilde{R}_i], E[\tilde{g}_i]),$$

or this function:

$$P_{0i} = f(D_{0i}, \beta_i, \beta_{gi}).$$

Therefore, given the last function, the determination of the intrinsic value of a stock implies the following steps: (1) establishment of the economics exogenous variables (common to every stock); (2) observation of the stock current dividend; and (3) estimation of stock's standard beta and stock's dividend growth beta.

The Estimation of Dividend Growth Beta

As we have already mentioned, the estimation of equilibrium dividend growth rates is not theoretically more difficult than estimating equilibrium returns. Additionally, estimating dividend growth betas is no more complicated than estimating standard betas. Moreover, it is easy to demonstrate that dividend growth betas could be approximated from standard betas, which from a very practical point of view, could be useful. Given the definition of betas and using the preceding assumption regarding the equivalence between the *expected growth rate in stock price* and the *expected dividend growth rate of the stock*, we can see that:

$$\beta_i = COV[\tilde{R}_m, \tilde{R}_i] / \sigma^2[\tilde{R}_m]$$

$$= \frac{COV[\tilde{g}_m + (1+\tilde{g}_m)D_{0m}/P_{0m}, \tilde{g}_i + (1+\tilde{g}_i)D_{0i}/P_{0i}]}{\sigma^2[\tilde{g}_m + (1+\tilde{g}_m)D_{0m}/P_{0m}]}$$

$$= \frac{COV[\tilde{g}_m + \tilde{g}_m D_{0m}/P_{0m} + D_{0m}/P_{0m}, \tilde{g}_i + \tilde{g}_i D_{0i}/P_{0i} + D_{0i}/P_{0i}]}{\sigma^2[\tilde{g}_m + \tilde{g}_m D_{0m}/P_{0m} + D_{0m}/P_{0m}]}$$

$$= \frac{COV[\tilde{g}_m(1+D_{0m}/P_{0m}), \tilde{g}_i(1+D_{0i}/P_{0i})]}{\sigma^2[\tilde{g}_m(1+D_{0m}/P_{0m})]}$$

$$= \frac{COV[\tilde{g}_m, \tilde{g}_i](1+D_{0m}/P_{0m})(1+D_{0i}/P_{0i})}{\sigma^2[\tilde{g}_m](1+D_{0m}/P_{0m})^2}$$

$$= \beta_{gi}(1+D_{0i}/P_{0i})/(1+D_{0m}/P_{0m}). \quad (27)$$

This means that standard betas are perfectly co-integrated with dividend growth betas, just as stock prices are perfectly co-integrated with dividends. This also means that an approximation of dividend growth betas can be performed using the following equivalence:

$$\beta_{gi} = \beta_i(1+D_{0m}/P_{0m})/(1+D_{0i}/P_{0i}). \quad (28)$$

Therefore, to estimate (or approximate) the dividend version of a particular beta, financial analysts are only required to know the market dividend-price ratio (D_{0m}/P_{0m}), the current dividend-price ratio of the stock (D_{0i}/P_{0i}), and its standard beta.

In addition, equation (28) suggests that dividend-growth beta could represent a measure of risk, if we recognize that standard beta is a fundamental risk measure according to the CAPM, and if we if accept that the dividend-price ratio (or its inverse) is sometime presented as a proxy of risk. In this sense, a risky stock should exhibit a low dividend-price ratio (see the denominator), a high standard beta (see the numerator), and a high dividend growth beta.

Consequently, we can also suggest that equation (21) represents a description of the relationship between dividend growth and risk. More particularly, we can suggest that the equilibrium expected dividend growth rate of a stock is linearly and positively related with risk, measured by its dividend growth beta.

As mentioned by Bergeron (2013-a) and Bergeron (2013-b), the relationship between dividend growth and risk (in its standard form) is not a new subject in finance. In fact, it is generally accepted that big old firms that already pay generous dividends and have low risk present low expected dividend growth in the long-run. Furthermore, many empirical studies indicate a positive correlation between dividend growth and risk (see Bergeron 2013-b, page 194 for example). The belief can be explained in this way: if firms are risk averse and cautious, then those operating at a high level of risk will wait to pay dividends to have enough retained earnings for bad earnings years. As a result, in a highly uncertain context, firms will simultaneously display a high measure of risk, a low current dividend (relative to earnings and future dividends), and a high expected dividend growth rate.

Market and zero-beta portfolios

The estimate for the expected dividend growth rate for the market portfolio could be as simple as the corresponding expected return. To estimate this value, we can simply choose a market index, calculate its historical average and then extrapolate the past into the future. For example, from 1941 to 2011 the average annual dividend growth rate of the S&P 500 was 6%.[6] However, as noted by Hurley and Johnson (1994), this procedure is clearly "rough and ready" and analysts would most likely have made appropriate adjustments to the data to suit their own expectations.

In the same manner, to estimate the expected dividend growth rate of the zero-beta portfolio, we can choose stocks that present very low betas and compute their average dividend growth rate. For instance, on November 2013, the standard beta of Southern Company (SO) was equal to 0.02, and its

[6] Source: Investment Research Series, Why Dividends Matter; gafunds.com.

corresponding average dividend growth rate was 4%.[7] We can also anticipate that the expected dividend growth rate for the zero-beta portfolio should be at least equivalent to the inflation rate, which was near 4% on average, from 1941 to 2011 (see, again, note 6).

4. Practical Application: an example

As noted by Hurley and Johnson (1994), dividend valuation models are obviously most useful for firms with a systematic pattern of dividend payout. To test their dividend models, they selected three telephone utilities, based on the assumption that these firms would have regular dividend payments. In the same vein, we estimate the stock value of Edison International (EIX).

To estimate the intrinsic value of Edison International, we followed Hurley and Johnson (see page 52) and first established that the required rate of return of a stock can be determined with the CAPM, using a historic market risk premium (λ_1) of 5% and a Treasury yield (λ_0) of 6%. With a standard beta of 0.50, for Edison International (see, again, note 7), this means that the required rate of return for the company was 8.50%.

Next, to estimate the expected dividend growth rate of the market portfolio we used the S&P 500 index, and to estimate the expected dividend growth rate of the zero-beta portfolio, we supposed that this value should be, at least, equivalent to the inflation rate. From 1941 to 2011 these values were respectively equal to 6% and 4%, we therefore determined that: $E[\tilde{g}_m] = 6\%$ and $E[\tilde{g}_z] = 4\%$, for a corresponding spread of 2%.

On December 1, 2014, the price of Edison International (Prev Close) was $63.56, and the annual dividend was $1.42, for a dividend yield ratio of 2.23%. At the same time, the dividend yield of the S&P 500 was 1.87%. Thus, from equation (28) the dividend growth beta was 0.4982 (0.50 x 1.0187 ÷ 1.0223), and the corresponding equilibrium dividend growth rate was 5.00%.

As a result, the intrinsic value for Edison International (*EIX*) was equal to $40.57, as shown below:

$$V_{0(EIX)} = \frac{D_{1(EIX)}}{E[\tilde{R}_{EIX}] - E[\tilde{g}_{EIX}]} = \frac{1.42}{0.0850 - 0.0500} = 40.57$$

with

$$E[\tilde{R}_{EIX}] = E[\tilde{R}_z] + (E[\tilde{R}_m] - E[\tilde{R}_z])\beta_{(EIX)} = 0.06 + (0.05)0.5000 = 0.0850$$

$$E[\tilde{g}_{EIX}] = E[\tilde{g}_z] + (E[\tilde{g}_m] - E[\tilde{g}_z])\beta_{g(EIX)} = 0.04 + (0.02)0.4982 = 0.0500$$

where $V_{0(EIX)}$, $D_{1(EIX)}$, $E[\tilde{R}_{EIX}]$, $\beta_{(EIX)}$, $\beta_{g(EIX)}$ and $E[\tilde{g}_{EIX}]$ respectively represent the intrinsic value, next dividend, required return, standard beta, dividend growth beta, and the equilibrium dividend growth rate of Edison International.

Consequently, the intrinsic value for Edison International is clearly inferior to its actual price. However, it is interesting to note that the price of the stock was 44.80 $ exactly one year before.

5. Conclusion

From the main implication of the CAPM, we developed a theoretical stock valuation model that takes into account the equilibrium dividend growth rate. Our first result showed that the expected dividend

[7] Source: http://finance.yayoo.com/.

growth rate of a stock is linearly and positively related to its dividend growth beta. This result was derivate without specific assumption – aside from the implicit assumption that stock prices and dividends are positive. The dividend growth beta parameter presented here is particularly interesting because of its similarity to the concept of cash-flow beta which has recently been proposed in the literature. Next, assuming that the expected dividend growth rate of a stock is equivalent to its capital gain rate, we proposed to employ our equilibrium relationship to estimate stock intrinsic values. Finally, we presented a practical application of our model, using a utility company. This simple example shows that our model could be used as a helpful tool for stock valuation.

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Appendix A: Dividend Growth and Dividend Growth Beta

In this appendix, we show that the expected dividend growth rate of a stock is positively related to its dividend growth beta. From equation (1), we can write that:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{\sigma^2[\tilde{R}_m]} COV[\tilde{R}_m, \tilde{R}_i] \qquad \text{(A1)}$$

or:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{R}_i] \qquad \text{(A2)}$$

where $K \equiv \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{\sigma^2[\tilde{R}_m]}$.

Also, from the definitions of return, capital gain and dividend yield, we can write that:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + KCOV[\tilde{G}_m + \tilde{d}_m, \tilde{G}_i + \tilde{d}_i]. \qquad \text{(A3)}$$

The proprieties of covariance allow us to establish that:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{G}_i] + KCOV[\tilde{G}_m, \tilde{d}_i] + KCOV[\tilde{d}_m, \tilde{d}_i]. \qquad \text{(A4)}$$

Now, if we note d_i as the courant dividend-price ratio for stock i ($d_i \equiv D_{0i} / P_{0i}$), and d_m as the corresponding value for the market ($d_m \equiv D_{0m} / P_{0m}$), we can express the relationship in this manner:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{G}_i] + KCOV[\tilde{G}_m, \tilde{d}_i] + d_m d_i KCOV[\tilde{g}_m, \tilde{g}_i]. \quad \text{(A5)}$$

Multiplying each side by $\sigma^2[\tilde{g}_m]$, we get:

$$E[\tilde{R}_i] = E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{G}_i] + KCOV[\tilde{G}_m, \tilde{d}_i] + \sigma^2[\tilde{g}_m] d_m d_i K\beta_{gi}. \quad \text{(A6)}$$

From the definition of return, we have:

$$\begin{aligned} &E[\tilde{G}_i] + E[1 + \tilde{g}_i] d_i \\ &\quad = E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{G}_i] + KCOV[\tilde{G}_m, \tilde{d}_i] + \sigma^2[\tilde{g}_m] d_m d_i K\beta_{gi}. \end{aligned} \quad \text{(A7)}$$

This leads us to isolate the expected dividend growth rate, to obtain:

$$\begin{aligned} E[\tilde{g}_i] &= (E[\tilde{R}_z] + KCOV[\tilde{R}_m, \tilde{G}_i] + KCOV[\tilde{G}_m, \tilde{d}_i] - E[\tilde{G}_i])/d_i \\ &\quad + \sigma^2[\tilde{g}_m] d_m K\beta_{gi} - 1. \end{aligned} \quad \text{(A8)}$$

As a result, we have:

$$\frac{\partial E[\tilde{g}_i]}{\partial \beta_{gi}} = \sigma^2[\tilde{g}_m] d_m K = \sigma^2[\tilde{g}_m] d_m \frac{E[\tilde{R}_m] - E[\tilde{R}_z]}{\sigma^2[\tilde{R}_m]} > 0.$$

The relationship is positive.