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Consumption, Residual Income Valuation, and Long-run Risk

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Consumption, Residual Income Valuation, and Long-run Risk

Abstract

This paper develops a theoretical extension of the residual income valuation model that integrates the concept of long-run risk. The model starts with an intertemporal framework, assumes the clean surplus accounting relation, and expresses firm market value as the book value of equity plus the present value of expected future residual income. The main finding of the extension model indicates that a firm's goodwill is negatively related to its accounting risk, measured by the long-run covariance of the firm's abnormal earnings growth and aggregate consumption growth. In the context of the residual income valuation method, this finding suggests that the earnings-consumption covariance (in the long run) represents an appropriate accounting risk measurement of a firm's intrinsic value.

Keywords: residual income valuation, long-run risk, consumption, accounting beta, intertemporal model.

1 Introduction

According to Beynon and Clatworthy (2013), the residual income valuation model is widely used by academic researchers and by investment practitioners. The model has been argued to be easier to employ and more accurate than those based on cash flows or dividends. In addition, the model is consistent with Miller and Modigliani's dividend irrelevance theorem (1961).

The residual income valuation model expresses a company's fundamental value as the sum of its book value and the present value of its expected future residual income.[1] This approach has its origins in Edwards and Bell (1961) and Peasnell (1981, 1982) and was popularized by many other researchers. For example, Ohlson (1995) extended the residual income model, assuming that abnormal earnings follow an autoregressive process of order 1.[2] The assumptions of the model lead to a linear, closed-form, valuation solution explaining goodwill.[3] Feltham and Ohlson (1999) provided a general version of the preceding model by introducing risk and stochastic interest rates. Baginski and Wahlen (2003) constructed a new accounting measure of the effect of risk on share price using the residual income model and risk-free rates of return. This new measure, which they called the *price differential*, captured the magnitude of the discount for risk implicit in share prices. Applying a scheme similar to the residual income method, Ohlson and Juettner-Nauroth (2005) developed a parsimonious model relating a firm's price to its cost-of-equity capital and growth in earnings. Nekrasov and Shroff (2009) employed the residual income model to analytically derive a risk

[1] Residual income represents the economic profit of the business after deducting the cost of capital.
[2] Here, the terms *residual income* and *abnormal earnings* are interchangeable.
[3] Goodwill equals market value minus book value.

adjustment that equals the covariance between a firm's return on book equity and economy-wide factors. Their approach separately estimated two components of value: risk-free present value and covariance risk adjustment. More recently, Beynon and Clatworthy (2013) introduced a fuzzy-based method that reflects the imprecision inherent in certain standard parameters of equity valuation.[4]

In this paper, we develop a theoretical extension of the residual income valuation model that takes into account the concept of long-run risk recently proposed in the literature.

Development of the model starts with the fundamental precept that each agent maximizes its utility function, and with the basic principle that the present value of expected dividends determines equity value. We then postulate that accounting data and dividends satisfy the clean surplus relation and express the market value of a firm as the sum of its book value per share and the present value of its corresponding future residual income. Next, we establish that a company's goodwill and the corresponding abnormal earnings are cointegrated. We demonstrate that the expected growth rate of the residual income is positively related to the covariance between abnormal earnings and aggregate consumption. This allows us to isolate the equilibrium equity value of a firm for a single period. Finally, we extend these results to several periods.

For a specific period, our model indicates that the intrinsic value of a firm equals its book value of equity plus an additional amount positively related to its abnormal earnings growth rate and negatively related to its *accounting consumption beta*, obtained from the covariance between firm's abnormal earnings and aggregate consumption. To integrate the long-run concept of risk over several periods, we extend (to earnings) the definition of long-run risk presented in Bansal et al. (2005), where risk is estimated by the long-run covariance between dividends and aggregate consumption (*cash flow beta*).[5] More precisely, over several periods our model reveals that the intrinsic value of a firm is negatively related to the *accounting cash flow beta*, measured by the long-run covariance between abnormal earnings and aggregate consumption.

The concept of accounting beta as a measure of risk represents an important research area in accounting and economics. Beaver et al. (1970) revealed a positive

[4] According to Beynon and Clatworthy (2013), a fuzzy environment is associated with human imprecision and vagueness, which is distinct from randomness.
[5] According to Beeler and Campbell (2012), the long-run risk models of Bansal and Yaron (2004) and Bansal et al. (2005) have attracted a great deal of attention, with important subsequent work by Hansen et al. (2008), Bansal et al. (2009), Bansal and Kiku (2011), and Bansal and Shaliastovich (2013), among others.

correlation between accounting betas and standard betas.[6] This observation was confirmed by Beaver and Manegold (1975) and Ismail and Kim (1989) under a variety of accounting return variables. In addition, Karels and Sackley (1993) examined the statistical relationship between market and accounting betas in the U.S. banking industry and concluded that accounting betas are correlated with their market counterparts at levels similar to other non-banking studies. Likewise, Baginski and Wahlen (2003) showed that accounting betas are significantly and positively related to *priced risk premiums* in univariate regressions. Furthermore, Nekrasov and Shroff's (2009) empirical observations validated their fundamental accounting risk measures, defined by the covariance between a firm's return on book equity and economy-wide risk factors. Da (2009) used the covariance between long-run accounting return and long-run aggregate consumption to explain the cross section of expected asset returns. The results indicate that the long-run earnings-consumption covariance explains more than 56 % of the cross-sectional variation in risk premia. As Goyal (2012, p. 29) mentioned, Da's model is useful in explaining returns. Also, as Ball and Sadka (2015, p. 51) pointed out, studies on systematic earnings risk represent a promising avenue for further research.

Nevertheless, none of the above-mentioned works develops a theoretical extension of the residual income model that explicitly integrates the accounting consumption beta or the accounting cash flow beta directly into the intrinsic value.

In this regard, our study is motivated by the following observations: (1) the importance of the residual income valuation method, and the accounting beta, in the accounting research area; (2) the recent success of the long-run concept of risk, in the literature;[7] (3) the empirical validity of the long-run earnings-consumption covariance for estimating risk;[8] (4) the absence of a theoretical model that explicitly integrates an accounting long-run risk measure in a residual income valuation context.

There is an enormous quantity of empirical studies related to stock or equity valuation. Similarly to the contributions of Ohlson (1995), Feltham and Ohlson (1995, 1999), Ohlson and Juettner-Nauroth (2005), or Bergeron (2013a, and 2013b) our contribution in the present paper is essentially theoretical. More precisely, the primary contribution of our study is to characterise the relationship between the earnings-consumption covariance and the firm theoretical value, using a residual income

[6] In Beaver et al. (1970), the accounting beta represents an estimation of the covariance between a firm's earnings and market earnings.

[7] As noted before, the long-run concept of risk has recently attracted a great deal of attention. Moreover, according to Ferson, Nallareddy and Xie (2013), the long-run model following Bansal and Yaron (2004) has been a phenomenal success.

[8] As mentioned above, the empirical results of Da (2009) indicate that the long-run earnings-consumption covariance explains more than 56 % of the cross-sectional variation in risk premia. If a duration measure is added, the model explains more than 80% (see also Goyal, 2012).

valuation approach. Moreover, our work suggests that the accounting cash flow beta represents an appropriate measure of risk (on the long-run). Furthermore, from a theoretical point of view, our model supports the use of accounting variables in estimating risk.

The remainder of the paper proceeds as follows. Section 2 presents the intertemporal equilibrium framework (the economy). Section 3 derives the equilibrium equity value of a firm for one period. Section 4 extends the derivation over many periods and links the intrinsic value of the firm to long-run risk. Section 5 provides the paper's conclusion.

2 The economy

In accordance with Bergeron (2013b, p. 187)[9], the intertemporal framework of our model assumes a restrictive economy in which the representative agent maximizes the time-separable utility function:

$$E_t \sum_{s=0}^{\infty} \delta^s U(\tilde{C}_{t+s}), \tag{1}$$

subject to constraints, where δ is the time discount factor ($0 < \delta < 1$), $U(\bullet)$ is an increasing concave and derivable function, and $\tilde{C}_{t+s}$ is the aggregate consumption at time $t+s$ ($s = 0, 1, 2, ..., \infty$).[10] The result of this problem leads us to the general representation of the dividend discount formula:[11]

$$V_{it} = E_t \sum_{s=1}^{\infty} \delta^s \frac{U'(\tilde{C}_{t+s})}{U'(C_t)} \tilde{D}_{i,t+s}, \tag{2}$$

where V_{it} represents the equity market value of firm *i* at time *t* and $\tilde{D}_{i,t+s}$ represents the dividends of firm *i* at time $t+s$ ($s = 1, 2, ..., \infty$).[12] Equation (2) indicates that the equity market value of a firm corresponds to the present value of all future cash flows (dividends), where the stochastic discount factor is equivalent to $\delta^s U'(\tilde{C}_{t+s}) / U'(C_t)$. To facilitate the estimation of Equation (2), we refer, like Bansal and Kiku (2011) and many others, to the standard assumption of a constant relative risk aversion via the power utility function given by $U(\tilde{C}_{t+s}) = \tilde{C}_{t+s}^{1-\gamma} / (1-\gamma)$, for $s = 0, 1, 2, ..., \infty$, where γ (

[9] See also Bergeron (2013a), and Bergeron et al. (2015).
[10] The operators E_t, VAR_t, and COV_t refer respectively to mathematical expectations, variance, and covariance, where index *t* implies that we consider the available information at time *t*.
[11] See Rubinstein (1976) or Cochrane (2005), Chapter 1.
[12] The premium (U') is a derivative of a function.

γ > 0) is the coefficient of relative risk aversion. With this assumption, the equilibrium market value becomes:

$$V_{it} = E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \tilde{D}_{i,t+s} . \tag{3}$$

The clean surplus relation establishes that: $\tilde{Y}_{i,t+s} = \tilde{Y}_{i,t+s-1} + \tilde{X}_{i,t+s} - \tilde{D}_{i,t+s}$, where $\tilde{Y}_{i,t+s}$ represents the book value of firm *i* at time $t+s$, $\tilde{Y}_{i,t+s-1}$ represents the book value of firm *i* at time $t+s-1$, and $\tilde{X}_{i,t+s}$ represents the earnings of firm *i* at time $t+s$ (for $s = 0, 1, 2, ..., \infty$)[13]. Assuming the clean surplus relation, we can write:

$$V_{it} = E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} (\tilde{Y}_{i,t+s-1} + \tilde{X}_{i,t+s} - \tilde{Y}_{i,t+s}) . \tag{4}$$

If we define the abnormal earnings of firm *i* at time $t+s$, $\tilde{X}^a_{i,t+s}$, in the following manner: $\tilde{X}^a_{i,t+s} \equiv \tilde{X}_{i,t+s} - R_{F,t+s}\tilde{Y}_{i,t+s-1}$, where $R_{F,t+s}$ represents the risk-free return between time $t+s-1$ and $t+s$ (for $s = 0, 1, 2, ..., \infty$); after algebraic manipulations, the residual income valuation model suggests that:[14]

$$V_{it} = Y_{it} + E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \tilde{X}^a_{i,t+s} . \tag{5}$$

Equation (5) indicates that the equity market value of a firm is equal to its book value plus the present value of all future abnormal earnings.

Equation (5) also indicates that the difference between the equity value of the firm and its book value or, if we prefer, its goodwill, corresponds to the present value of all future abnormal earnings, that is:

$$S_{it} = E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \tilde{X}^a_{i,t+s} , \tag{6}$$

[13] Given the available information at time *t*, notice that if the value of the index *s* is equal to zero, then the corresponding variable is not a random variable, and the tilde (~) should not appear.
[14] See, for example, Nekrasov and Shroff (2009, p. 1987).

where S_{it} represents the goodwill (*the spread*) of firm *i* at time *t* ($S_{it} = V_{it} - Y_{it}$). Since the current abnormal earnings of firm *i* at time *t*, X_{it}^{a}, are known given the available information at time *t*, the constant value can thus be passed through the conditional expectation operator of Equation (6) to indicate:

$$S_{it} = X_{it}^{a} E_t \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \frac{\tilde{X}_{i,t+s}^{a}}{X_{it}^{a}}, \tag{7}$$

or, to simplify the notation:

$$S_{it} = X_{it}^{a} E_t[\tilde{Z}_{it}], \tag{8}$$

where the random variable $\tilde{Z}_{it}$ is defined in this manner: $\tilde{Z}_{it} \equiv \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+s}}{C_t} \right)^{-\gamma} \frac{\tilde{X}_{i,t+s}^{a}}{X_{it}^{a}}$.

To further simplify the notation, we can also write:

$$S_{it} = X_{it}^{a} \mu_{it}, \tag{9}$$

where the parameter μ_{it} corresponds to the mathematical expectation of the random variable $\tilde{Z}_{it}$, given the available information at time *t*, that is to say: $\mu_{it} \equiv E_t[\tilde{Z}_{it}]$. In the same way, given the available information at time $t+1$, we can determine that the equity market value of firm *i* at time *t*+1 ($V_{i,t+1}$) minus its corresponding book value ($Y_{i,t+1}$) equals its corresponding goodwill ($S_{i,t+1}$), and that:

$$S_{i,t+1} = X_{i,t+1}^{a} E_{t+1}[\tilde{Z}_{i,t+1}], \tag{10}$$

where $X_{i,t+1}^{a} \equiv X_{i,t+1} - R_{F,t+1} Y_{it}$, with $\tilde{Z}_{i,t+1} \equiv \sum_{s=1}^{\infty} \delta^s \left(\frac{\tilde{C}_{t+1+s}}{C_{t+1}} \right)^{-\gamma} \frac{\tilde{X}_{i,t+1+s}^{a}}{X_{i,t+1}^{a}}$.

Moreover, if the random variables $\tilde{Z}_{it}$ and $\tilde{Z}_{i,t+1}$ are independent and identically distributed (*i.i.d.*), then given the available information at time *t*, we can assert that the random goodwill of firm *i* at time *t*+1 ($\tilde{S}_{i,t+1}$) is stochastically related to earnings in this manner:

$$\tilde{S}_{i,t+1} = \tilde{X}^{a}_{i,t+1}\mu_{it}\,. \tag{11}$$

where $\tilde{X}^{a}_{i,t+1}$ represents the random abnormal earnings of firm *i* at time *t*+1.

To summarize, Equation (11) suggests that goodwill and abnormal earnings are cointegrated. This result allows us to simplify the expression given by the standard residual income model without necessarily assuming constant growth over time for the abnormal earnings.

3 Intrinsic value for one period

Based on the above equilibrium conditions, we derive, in this section, the intrinsic equity value of a firm for one period. Our derivation assumes that the abnormal earnings growth rate and aggregate consumption growth rate have a bivariate normal distribution.[15]

Recursively, Equation (3) can be expressed for a single period in the following standard manner:

$$V_{it} = E_t[\tilde{M}_{t+1}(\tilde{V}_{i,t+1} + \tilde{D}_{i,t+1})], \tag{12}$$

where $\tilde{M}_{t+1}$ corresponds to the intertemporal marginal rate of substitution between t and $t+1$, that is: $\tilde{M}_{t+1} \equiv \delta U'(\tilde{C}_{t+1})/U'(C_t)$. In the same manner, recursively from Equation (6), we can write:

$$S_{it} = E_t[\tilde{M}_{t+1}(\tilde{S}_{i,t+1} + \tilde{X}^{a}_{i,t+1})]\,. \tag{13}$$

Substituting Equation (9) and Equation (11) into Equation (13) shows that:

$$X^{a}_{it}\mu_{it} = E_t[\tilde{M}_{t+1}(\mu_{it}\tilde{X}^{a}_{i,t+1} + \tilde{X}^{a}_{i,t+1})]. \tag{14}$$

After simple manipulations, we can deduce that:

[15] In Appendix A, we derive the intrinsic value of a firm for one period without specific probability distribution.

$$1 = E_t[\tilde{M}_{t+1}(1+\tilde{g}^a_{i,t+1})(1+1/\mu_{it})] \tag{15}$$

where $\tilde{g}^a_{i,t+1}$ represents the growth rate of the abnormal earnings between time t and t + 1, that is: $\tilde{g}^a_{i,t+1} \equiv \tilde{X}^a_{i,t+1}/X^a_{it} - 1$. Based on Equation (12) and the risk-free rate of return, we know that:

$$1 = E_t[\tilde{M}_{t+1}(1+R_{F,t+1})]. \tag{16}$$

After simple manipulations, Equation (15) minus Equation (16) reveals the following equilibrium condition:

$$0 = E_t[\tilde{M}_{t+1}\{(1+\tilde{g}^a_{i,t+1})(1+1/\mu_{it}) - (1+R_{F,t+1})\}]. \tag{17}$$

Using the mathematical definition of covariance, the last equality implies that:

$$\begin{aligned} &COV_t[\tilde{M}_{t+1}, (1+\tilde{g}^a_{i,t+1})(1+1/\mu_{it}) - (1+R_{F,t+1})] \\ &= -E_t[\tilde{M}_{t+1}]E_t[(1+\tilde{g}^a_{i,t+1})(1+1/\mu_{it}) - (1+R_{F,t+1})], \end{aligned} \tag{18}$$

and covariance properties show that:

$$\begin{aligned} &COV_t[\tilde{M}_{t+1}, 1+\tilde{g}^a_{i,t+1}](1+1/\mu_{it}) \\ &= -E_t[\tilde{M}_{t+1}]E_t[1+\tilde{g}^a_{i,t+1}](1+1/\mu_{it}) + E_t[\tilde{M}_{t+1}](1+R_{F,t+1}). \end{aligned} \tag{19}$$

From Equation (16), we have: $E_t[\tilde{M}_{t+1}] = 1/(1+R_{F,t+1})$, which indicates that:

$$\begin{aligned} &COV_t[\tilde{M}_{t+1}, 1+\tilde{g}^a_{i,t+1}](1+1/\mu_{it}) \\ &= -E_t[1+\tilde{g}^a_{i,t+1}](1+1/\mu_{it})/(1+R_{F,t+1}) + 1. \end{aligned} \tag{20}$$

Therefore, in equilibrium, the expected abnormal earnings growth rate of any firm is such that:

$$E_t[1+\tilde{g}^a_{i,t+1}] = (1+R_{F,t+1})/(1+1/\mu_{it}) - (1+R_{F,t+1})COV_t[\tilde{M}_{t+1}, \tilde{g}^a_{i,t+1}]. \tag{21}$$

From the definition of the utility function, the marginal rate of substitution between t and $t+1$ becomes: $\tilde{M}_{t+1} = \delta(1+\tilde{g}_{t+1})^{-\gamma}$, where $\tilde{g}_{t+1}$ represents the aggregate consumption growth rate between time t and t+1 ($\tilde{g}_{t+1} \equiv \tilde{C}_{t+1}/C_t - 1$). Equation (21) therefore becomes:

$$E_t[1+\tilde{g}^a_{i,t+1}] = \frac{1+R_{F,t+1}}{1+1/\mu_{it}} - (1+R_{F,t+1})COV_t[\delta(1+\tilde{g}_{t+1})^{-\gamma}, \tilde{g}^a_{i,t+1}]. \qquad (22)$$

With the available information at time t, the last relationship offers a simple equilibrium condition for which the only two random variables are given by growth rates—the growth rate of aggregate consumption and the growth rate of abnormal earnings. To simplify the application and comprehension of this relationship, we suppose (based on Rubinstein 1976) that the abnormal earnings of the firm and the aggregate consumption have a bivariate normal distribution. In this manner, based on Stein's lemma, we can rewrite Equation (22) in the following form:[16]

$$E_t[1+\tilde{g}^a_{i,t+1}] = (1+R_{F,t+1})/(1+1/\mu_{it})$$
$$-(1+R_{F,t+1})\delta(-\gamma)E_t[(1+\tilde{g}_{t+1})^{-\gamma-1}]COV_t[\tilde{g}_{t+1}, \tilde{g}^a_{i,t+1}]. \qquad (23)$$

Multiplying on each side by the conditional variance of the aggregate consumption growth rate, $\sigma^2_t[\tilde{g}_{t+1}]$, shows that:

$$E_t[1+\tilde{g}^a_{i,t+1}] = (1+R_{F,t+1})/(1+1/\mu_{it})$$
$$-(1+R_{F,t+1})\delta(-\gamma)E_t[(1+\tilde{g}_{t+1})^{-\gamma-1}]\sigma^2_t[\tilde{g}_{t+1}]COV_t[\tilde{g}_{t+1}, \tilde{g}^a_{i,t+1}]/\sigma^2_t[\tilde{g}_{t+1}], \qquad (24)$$

or to simplify the expression:

$$E_t[1+\tilde{g}^a_{i,t+1}] = (1+R_{F,t+1})/(1+1/\mu_{it}) + \lambda_t\beta^a_{it}, \qquad (25)$$

where

$$\lambda_t \equiv -(1+R_{F,t+1})\delta(-\gamma)E_t[(1+\tilde{g}_{t+1})^{-\gamma-1}]\sigma^2_t[\tilde{g}_{t+1}] > 0,$$

$$\beta^a_{it} \equiv COV_t[\tilde{g}_{t+1}, \tilde{g}^a_{i,t+1}]/\sigma^2_t[\tilde{g}_{t+1}].$$

[16] If x and y have a bivariate normal distribution, $COV(y, f(x)) = E(f'(x))COV(y,x)$. See Huang and Litzenberger (1989, p. 101).

Equation (25) represents a basic equilibrium condition expressed with abnormal earnings growth rates. The condition implies that a firm's expected abnormal earnings growth rate is linearly related to the covariance between its abnormal earnings and aggregate consumption. More particularly, parameter β_{it}^{a} in Equation (25) represents the *accounting consumption beta* of firm *i* at time *t*, defined by the covariance between its abnormal earnings growth rate and the aggregate consumption growth rate, divided by the variance of the aggregate consumption growth rate.[17] It measures how sensitive a company's abnormal earnings are to aggregate consumption (reflecting economic activity). Parameter λ_t defines the direction and inclination of the relationship. Its value is positive $(\lambda_t > 0)$ because the value $\tilde{C}_{t+s} / C_t = 1 + \tilde{g}_{t+1}$ is superior to zero, since $\tilde{C}_{t+1}$ and C_t are, by definition positive, and because all the following values, $R_{F,t+1}$, δ, and γ, including in the parameter, are positive by construction. In short, the equilibrium condition expressed by Equation (25) indicates that a firm's expected abnormal earnings growth is linearly and positively related to its accounting consumption beta.

Based on the above relationship, we can easily find a firm's intrinsic equity value. Isolating the term $1/\mu_{it}$ gives:

$$1/\mu_{it} = \frac{1 + R_{F,t+1}}{E_t[1 + \tilde{g}_{i,t+1}^{a}] - \lambda_t \beta_{it}^{a}} - 1, \tag{26}$$

or, if we prefer:

$$1/\mu_{it} = \frac{1 + R_{F,t+1}}{E_t[1 + \tilde{g}_{i,t+1}^{a}] - \lambda_t \beta_{it}^{a}} - \frac{E_t[1 + \tilde{g}_{i,t+1}^{a}] - \lambda_t \beta_{it}^{a}}{E_t[1 + \tilde{g}_{i,t+1}^{a}] - \lambda_t \beta_{it}^{a}}. \tag{27}$$

After simple manipulations, we get:

$$\mu_{it} = \frac{E_t[1 + \tilde{g}_{i,t+1}^{a}] - \lambda_t \beta_{it}^{a}}{R_{F,t+1} + \lambda_t \beta_{it}^{a} - E_t[\tilde{g}_{i,t+1}^{a}]}. \tag{28}$$

In accordance with Equation (9), we know that: $\mu_{it} = S_{it} / X_{it}^{a}$. Therefore, by the definition of firm goodwill, we have:

[17] Given the available information at time *t*, if the earnings-consumption covariance is positive (negative), then the corresponding covariance between abnormal earnings and aggregate consumption is also positive (negative).

$$V_{it} - Y_{it} = \frac{E_t[1+\tilde{g}^a_{i,t+1}] - \lambda_t \beta^a_{it}}{R_{F,t+1} + \lambda_t \beta^a_{it} - E_t[\tilde{g}^a_{i,t+1}]} X^a_{it}. \quad (29)$$

Equation (29) indicates that the amount of a firm's goodwill is a function of its current abnormal earnings, abnormal earnings growth rate for a single period, and accounting consumption beta. We can therefore establish that:

$$V_{it} = Y_{it} + \frac{E_t[1+\tilde{g}^a_{i,t+1}] - \lambda_t \beta^a_{it}}{R_{F,t+1} + \lambda_t \beta^a_{it} - E_t[\tilde{g}^a_{i,t+1}]} X^a_{it}. \quad (30)$$

Equation (30) represents our first result concerning intrinsic value. The equation reveals that the intrinsic value of a firm equals its book value plus an additional amount directly proportional to its current abnormal earnings, positively related to its abnormal earnings growth rate and negatively related to its accounting consumption beta.

To evaluate the intrinsic equity value of a firm based on Equation (30), we must: 1) establish the appropriate free rate of return using a standard fixed income instrument, 2) find the current book value and current earnings based on the firm's financial statements and calculate the corresponding abnormal earnings, 3) establish the firm's expected abnormal earnings growth rate based on our firm's forecast, 4) estimate the firm's accounting consumption beta using aggregate consumption predictions, and 5) determine the common parameter λ_t. Steps one to three are familiar. In theory, step four should not be more difficult or complicated than estimating the standard consumption beta or the usual accounting beta. Step five, on the order hand, requires knowledge of the representative investor's complete utility function. More specifically, to estimate the parameter λ_t we need to know the time discount factor value (δ) and the relative risk aversion coefficient value (γ), in addition to the following values: $R_{F,t+1}$, $E_t[(1+\tilde{g}_{t+1})^{-\gamma-1}]$, and $\sigma^2_t[\tilde{g}_{t+1}]$.

To facilitate the estimation of the parameter λ_t, we can also use the market portfolio, grouping together all companies in the economy.

Based on Equation (25), we can write:

$$E_t[1+\tilde{g}^a_{m,t+1}] = (1+R_{F,t+1})/(1+1/\mu_{mt}) + \lambda_t \beta^a_{mt}, \quad (31)$$

where the index *m* indicates the market portfolio. Rearranging provides the following value for our estimation:

$$\lambda_t = [E_t[1 + \tilde{g}^{a}_{m,t+1}] - (1 + R_{F,t+1}) / (1 + 1/\mu_{mt})] / \beta^{a}_{mt} , \quad (32)$$

knowing from Equation (9) that: $\mu_{mt} = S_{mt} / X^{a}_{mt}$.

As a result, if we already have an idea of the free-risk rate of return in the economy, the only estimated data we need to evaluate a firm's intrinsic equity value can be found in market portfolio financial data and firm financial data. Moreover, all parameters are relatively easy to interpret and present central values from which analysts can start its projections, and its estimates. For instance, the central value for the abnormal earnings growth rate of the firm (or the market portfolio) could provide from our observations of the corresponding historic growth rate for an appropriate index, while the central value of the *firm's accounting consumption beta* to *market accounting consumption beta* ratio should be one, knowing that this ratio equals one for the market portfolio (1 = β_m/β_m).

Regardless, a key point of interest of our residual income valuation extension is that we can easily use it to integrate the long-run concept of risk recently proposed in the literature.

4 Intrinsic value and long-run risk

According to Ferson et al. (2013), the long-run concept of risk has been a phenomenal success. Bansal and Yaron (2004) revealed that consumption and dividend growth rates include a small long-run component that explains key asset market phenomena. Bansal et al. (2005) showed that cash flow betas, obtained from the long-run covariance between dividend growth rate and aggregate consumption growth rate, account for more than 60% of the cross-sectional variation in risk premia. Parker and Julliard (2005) measured the risk of an asset by its ultimate risk to consumption and found that this risk measurement can largely explain the cross-sectional pattern of expected asset returns. Bansal et al. (2009) argued that the cointegrating relation between dividends and consumption represents a key determinant of risk premia across all investment horizons. Da (2009) proposed that the covariance between long-run accounting return and long-run aggregate consumption represents an appropriate measure of risk and showed that this measure of risk explains 58% of the cross-sectional variation in risk premia; when we add a duration measure, the model explains more than 80%. Along these lines, we can also mention the studies of Bansal and Kiku (2011), Bansal and Shaliastovich (2013), Bergeron (2013a, and 2013b), and Croce et al. (2015).[18]

[18] See also Hansen et al. (2008) and Bansal et al. (2009).

In this section, we integrate the long-run concept of risk into the residual income valuation model using the approach proposed by Bansal et al. (2005).

Accounting cash flow betas

According to Bansal et al. (2005), asset prices reflect the discounted value of cash flows, and changes in expected cash flows are an important ingredient in determining asset return news. Systematic risks in cash flows should therefore have some bearing on the risk compensation of assets, and assets with higher consumption risk cash flows (larger cash flow beta) should carry a higher risk premium. Starting from these implications, the authors examined their cash flow betas ability to explain the cross section of equity return portfolios. Their empirical estimation of their 30 characteristic-sorted portfolio cash flow betas confirmed their intuitions. The risk premium associated with long-run consumption risk was positive and highly significant when risk measures were obtained using the covariance between the growth rates of portfolio dividends (cash flows) and aggregate consumption growth rates in the long run. Motivated by the success of the long-run risk concept, Bergeron (2013a, 2013b) used the long-run covariance between dividend and consumption as the appropriate measure of risk for stock valuation. In the following development section, we will adapt this point of view to future earnings, as Da proposed (2009).

N periods

Our model development starts the economy at time *t*, but we can also start the economy at time *t*+1, time *t*+2, time *t*+3, etc. In so doing, our variable definitions and notation will be the same, except for the index of time. Given the available information at time *t*+*n* ($n = 0, 1, 2, ..., N-1$), we can assume that the representative agent maximizes the time-separable utility function:

$$E_{t+n} \sum_{s=0}^{\infty} \delta^{s} U(\tilde{C}_{t+n+s}), \tag{33}$$

where $\tilde{C}_{t+n+s}$ is the aggregate consumption at time $t+n+s$ ($s = 0, 1, 2, ..., \infty$). The result of this problem now lead us to the following dividend discount formula:

$$V_{i,\,t+n} = E_{t+n} \sum_{s=1}^{\infty} \delta^{s} \frac{U'(\tilde{C}_{t+n+s})}{U'(C_{t+n})} \tilde{D}_{i,\,t+n+s}, \tag{34}$$

where $V_{i,t+n}$ represents the equity market value of firm *i* at time *t*+*n*, and $\tilde{D}_{i,\,t+n+s}$ represents the dividends of firm *i* at time $t+n+s$. Assuming that the canonical

CCAPM of Rubinstein (1976), Lucas (1978), and Breeden (1979) is literally true, this means that:

$$1 = E_{t+n}[\tilde{\mathrm{M}}_{t+n+1}(1 + R_{F,\,t+n+1})], \tag{35}$$

where $R_{F,\,t+n+1}$ corresponds to the risk-free return between time $t+n$ and $t+n+1$, and where $\tilde{\mathrm{M}}_{t+n+1}$ (non-italicized) corresponds to the intertemporal marginal rate of substitution between time *t*+*n* and $t+n+1$ ($\tilde{\mathrm{M}}_{t+n+1} \equiv \delta U'(\tilde{C}_{t+n+1})/U'(C_{t+n})$).

If we keep the previous variable definitions (with the index of time *t*+*n* instead of *t*) from now on, in accordance with Equation (15), we can write that:

$$1 = E_{t+n}[\tilde{\mathrm{M}}_{t+n+1}(1 + \tilde{g}^{a}_{i,\,t+n+1})(1 + 1/\mu_{\mathrm{i},\,t+n})], \tag{36}$$

where $\tilde{g}^{a}_{i,\,t+n+1} \equiv \tilde{X}^{a}_{i,\,t+n+1}/X^{a}_{i,\,t+n} - 1$ and where $\mu_{\mathrm{i},\,t+n} \equiv E_{t+n}[\tilde{Z}_{i,\,t+n}]$.

Based on Equations (16) and (17), the equilibrium condition of the model now indicates that:

$$0 = E_{t+n}[\tilde{\mathrm{M}}_{t+n+1}\{(1 + \tilde{g}^{a}_{i,\,t+n+1})(1 + 1/\mu_{\mathrm{i},\,t+n}) - (1 + R_{F,\,t+n+1})\}]. \tag{37}$$

Taking the conditional mathematical expectation on each side of Equation (37) reveals that:

$$0 = E_t[E_{t+n}[\tilde{\mathrm{M}}_{t+n+1}\{(1 + \tilde{g}^{a}_{i,\,t+n+1})(1 + 1/\mu_{\mathrm{i},\,t+n}) - (1 + R_{F,\,t+n+1})\}]], \tag{38}$$

which allows us to ignore the full information set at time *t*+*n* and release the index *t*+*n* of the conditional operator in this manner:

$$0 = E_t[\tilde{\mathrm{M}}_{t+n+1}\{(1 + \tilde{g}^{a}_{i,\,t+n+1})(1 + 1/\mu_{\mathrm{i},\,t+n}) - (1 + R_{F,\,t+n+1})\}]. \tag{39}$$

From Equation (14) to Equation (21), with the marginal rate of substitution $\tilde{\mathrm{M}}_{t+n+1}$ (rather than $\tilde{M}_{t+1}$), we get:

$$E_t[1 + \tilde{g}^{a}_{i,\,t+n+1}] = \frac{1 + R_{F,\,t+n+1}}{1 + 1/\mu_{\mathrm{i},\,t+n}} - (1 + R_{F,\,t+n+1})COV_t[\tilde{\mathrm{M}}_{t+n+1}, \tilde{g}^{a}_{i,\,t+n+1}], \tag{40}$$

or (integrating the complete expression of the variable $\tilde{\mathrm{M}}_{t+n+1}$):

$$E_t[1+\tilde{g}^a_{i,t+n+1}] = \frac{1+R_{F,t+n+1}}{1+1/\mu_{i,t+n}} - (1+R_{F,t+n+1})COV_t\left[\frac{\delta U'(\tilde{C}_{t+n+1})}{U'(C_{t+n})}, \tilde{g}^a_{i,t+n+1}\right]. \quad (41)$$

Thus, from Equation (17) to Equation (25), we can express the following relationship (given the available information at time *t*):

$$E_t[1+\tilde{g}^a_{i,t+n+1}] = (1+R_{F,t+n+1})/(1+1/\mu_{i,t+n}) + \lambda_{t+n}\beta^a_{i,t+n}, \quad (42)$$

with

$$\lambda_{t+n} \equiv -(1+R_{F,t+n+1})\delta(-\gamma)E_t[(1+\tilde{g}_{t+n+1})^{-\gamma-1}]\sigma^2_t[\tilde{g}_{t+n+1}] > 0 \ ,$$

$$\beta^a_{i,t+n} \equiv COV_t[\tilde{g}_{t+n+1}, \tilde{g}^a_{i,t+n+1}]/\sigma^2_t[\tilde{g}_{t+n+1}],$$

where $\tilde{g}_{t+n+1}$ represents the aggregate consumption growth rate between time $t+n$ and time $t+n+1$.

In the long run (for *N* periods), the relationship between the company's earnings growth rate and its sensitivity to aggregate consumption can be obtained by summing from $n = 0$ to $n = N$-1:

$$\sum_{n=0}^{N-1} E_t[1+\tilde{g}^a_{i,t+n+1}] = \sum_{n=0}^{N-1}[(1+R_{F,t+n+1})/(1+1/\mu_{i,t+n}) + \lambda_{t+n}\beta^a_{i,t+n}]. \quad (43)$$

Assuming, in accordance with our previous assumptions and variable definitions, that $\mu_{i,t+n}$ is stationary ($\mu_{it} = \mu_{i,t+1} = \mu_{i,t+2}$, etc.), we can rewrite the summation operation in this way:

$$\sum_{n=0}^{N-1} E_t[1+\tilde{g}^a_{i,t+n+1}] = (1+1/\mu_{it})^{-1}\sum_{n=0}^{N-1}(1+R_{F,t+n+1}) + \sum_{n=0}^{N-1}\lambda_{t+n}\beta^a_{i,t+n}\ . \quad (44)$$

Multiplying by the scalar value $\sum_{n=0}^{N-1}\lambda_{t+n}$ on each side of Equation (44) shows that:

$$\sum_{n=0}^{N-1} E_t[1+\tilde{g}^a_{i,t+n+1}] = (1+1/\mu_{it})^{-1}\sum_{n=0}^{N-1}(1+R_{F,t+n+1}) + \sum_{n=0}^{N-1}\lambda_{t+n}\sum_{n=0}^{N-1} w_{t+n}\beta^a_{i,t+n}\ , \quad (45)$$

where $w_{t+n} \equiv \lambda_{t+n} / \sum_{n=0}^{N-1}\lambda_{t+n}$, with $0 < w_{t+n} < 1$. Dividing by N on each side of Equation (45) indicates that:

$$1+\bar{g}^a_{it} = (1+\bar{R}_{Ft})/(1+1/\mu_{it}) + \bar{\lambda}_t\bar{\beta}^a_{it}\ , \quad (46)$$

where

$$\bar{g}^a_{it} \equiv \sum_{n=0}^{N-1} E_t[\tilde{g}^a_{i,t+n+1}]/N\ ,\quad \bar{R}_{Ft} \equiv \sum_{n=0}^{N-1} R_{F,t+n+1}/N\ ,$$
$$\bar{\lambda}_t \equiv \sum_{n=0}^{N-1}\lambda_{t+n}/N\ ,\quad \bar{\beta}^a_{it} \equiv \sum_{n=0}^{N-1} w_{t+n}\beta^a_{i,t+n}.$$

Using this operation, the conditional estimator $\bar{g}^a_{it}$, $\bar{R}_{Ft}$ or $\bar{\lambda}_t$ can be viewed as an arithmetic average (over N periods), while the conditional estimator $\bar{\beta}^a_{it}$ can be viewed as a weighted average, where parameters w_{t+n} (n = 0, 1, 2, …, N-1) equivalent weight.

Equation (46) represents another basic equilibrium condition expressed with abnormal earnings growth rates. This time, the condition reveals that a firm’s long-run abnormal earnings growth rate is linearly related to the long-run covariance between its abnormal earnings and aggregate consumption. Particularly, parameter $\bar{\beta}^a_{it}$ in Equation (46) corresponds to the *accounting cash flow beta* of firm *i* at time *t*, measured by the long-run covariance between its abnormal earnings growth rate and aggregate consumption growth rate. The direction and inclination of the relationship is defined this time by the parameter $\bar{\lambda}_t$, which is also positive, because $\lambda_{t+1}, \lambda_{t+2}, \lambda_{t+3}$, etc., are all positive. As a result, the equilibrium condition expressed by Equation (46) now indicates that a firm’s expected abnormal earnings growth in the long run is linearly and positively related to its accounting cash flow beta. If the value of the index N is equal to one ($N-1=0$), then in this particular case, we recognize Equation (25) again, previously derived in Section 3.

After simple algebraic manipulations, we can isolate the conditional expectation μ_{it} to indicate that:

$$\mu_{it} = \frac{1 + \bar{g}_{it}^{a} - \bar{\lambda}_t \bar{\beta}_{it}^{a}}{\bar{R}_{Ft} + \bar{\lambda}_t \bar{\beta}_{it}^{a} - \bar{g}_{it}^{a}}, \tag{47}$$

and knowing that $\mu_{it} = (V_{it} - Y_{it}) / X_{it}^{a}$, we can express a firm's goodwill in this manner:

$$V_{it} - Y_{it} = \frac{1 + \bar{g}_{it}^{a} - \bar{\lambda}_t \bar{\beta}_{it}^{a}}{\bar{R}_{Ft} + \bar{\lambda}_t \bar{\beta}_{it}^{a} - \bar{g}_{it}^{a}} X_{it}^{a} \tag{48}$$

Here, Equation (48) indicates that a firm's goodwill is a function of its current abnormal earnings, average abnormal earnings growth rate (for *N* periods), and accounting cash flow beta. If we isolate the firm's value, we can finally establish that:

$$V_{it} = Y_{it} + \frac{1 + \bar{g}_{it}^{a} - \bar{\lambda}_t \bar{\beta}_{it}^{a}}{\bar{R}_{Ft} + \bar{\lambda}_t \bar{\beta}_{it}^{a} - \bar{g}_{it}^{a}} X_{it}^{a}. \tag{49}$$

Equation (49) represents our second result, concerning intrinsic value. The equation proposes that the intrinsic value of a firm equals its book value plus an additional amount directly proportional to its current abnormal earnings, positively related to its long-run abnormal earnings growth rate, and, this time, negatively related to its accounting cash flow beta.

Because the relationship between the value of the firm and its accounting cash flow beta is negative, the last parameter is viewed as a rightful measure of risk in the long run. Consequently, a firm's intrinsic value appears to be determined by its long-run accounting risk, estimated here by its accounting cash flow beta. In this sense, the above value expression supports the use of fundamental variables as earnings for estimating long-run risk in a residual income valuation context.

The valuation model derived in this section presents other interesting characteristics (from a theoretical or an empirical point of view). The model does not assume that dividends or earnings will grow at the same rate in the future or that the distribution ratio is fixed over time. To estimate the expected abnormal earnings growth rate in the long run, investors will have to make a prediction for each period and then compute the average. Investors could suppose, for example, that significant growth will continue for a certain period of time, after which growth will be normal. Similarly, the estimation of long-run risk will cover many periods, and the estimation of each periodic beta or periodic lambda could refer to a single-period estimation as described in Section 3. This multi-period procedure allows us to refine the valuation process and more importantly, to integrate the long-run concept of risk as proposed by Bansal et al.

(2005), knowing that their theoretical definition of risk supposes that cash flow growth rates vary over time (g is not fixed).[19] Moreover, our methodology has an important difference when compared with Bansal et al.'s model (2005). Our derivation and long-run risk definition are not limited by a restrictive linear approximation, as we can see from Bansal et al.'s first theoretical equation (2005, p. 1641).

In their study, Nekrasov and Shroff (2009) proposed a methodology that incorporates fundamental risk measures directly into the theoretical valuation model. This interesting model characteristic also appears in our model. Indeed, our framework explicitly integrates and identifies the measure of risk directly into the final valuation formula expressed by Equation (49). Furthermore, assuming the clean surplus accounting relation, our framework indicates that the measure of risk comes directly from different manipulations of the first-order condition of Rubinstein's (1976) fundamental economic problem (see our Equations (1) and (2)).

More important, according to Nekrasov and Shroff (2009, p. 1984), if firm intrinsic value is determined by fundamental economic variables such as earnings cash flows, then it makes sense to measure risk directly from earnings cash flows. Our valuation model is consistent with this premise. However, Nekrasov and Shroff's study (2009) never explicitly referred to the long-run concept of risk.

Beside, Da (2009) pointed out that long-run earnings-consumption covariance is able to explain cross-sectional variation in expected asset returns. In his study, risk measures were estimated in a novel way using long-run accounting earnings and aggregate consumption values exclusively. Our last valuation formula (expressed by Equation (49)) is consistent with these findings. In this sense, we can argue that our valuation formula represents a straightforward application of Da's recent empirical results and that our theoretical valuation model presents an interesting characteristic: the model utilizes a recent theoretical measure of systematic risk, empirically validated.

In addition, as noted by Da (2009), pinning risk to earnings cash flows instead of prices or returns has the following advantages. First, in the short term, price may temporarily deviate from its fair value due to mispricing or liquidity events. Second, in typical asset pricing models, prices and returns are set by expectations of future cash flows. Furthermore, measuring earnings and consumption in the long run alleviates problems caused by short-term earnings management and short-term consumption commitment. The valuation model derived in this section is also consistent with all of these characteristics.

[19] See Bansal et al. (2005, p. 1641–1642).

In the same manner, as Da (2009) noted, even if dividends and earnings are theoretically equivalent (using the clean surplus accounting relation), earnings data offers a better empirical estimation of future cash flows. For example, some firms are expected to pay no dividends for a long time into the future, and most firms tend to keep a stable dividend payout policy.[20] Taking this issue into consideration, we can also suppose that pinning risk to earnings instead of dividends (as Bansal et al. 2005; Hansen et al. 2008; and many others) increases the efficiency of risk estimation if the stock valuation estimation adopts an empirical point of view.[21]

5 Conclusion

There are many empirical studies related to stock or equity valuation. Our contribution is strictly theoretical. In this paper, we developed a theoretical extension of the residual income valuation model that integrated the novel concept of long-run risk. Using the intertemporal CCAPM framework and assuming the clean surplus accounting relation, we demonstrated that a firm's goodwill is negatively related to its long-run accounting risk. More precisely, we demonstrated that a firm's intrinsic value is equal to its book value, plus an additional amount negatively related to its accounting cash flow beta, obtained from the long-run earnings-consumption covariance. We argued that this accounting beta represents an appropriate accounting risk measure (in the long run). Overall, in the context of the residual income valuation method, our findings support the view that accounting risk defined over many periods affects the firm's theoretical value.[22]

The extension model presented here offers several interesting aspects in addition to the integration of the long-run risk concept. For example, the model indicates the abnormal earnings growth rate of a firm under equilibrium conditions. Additionally, the model's parameters are easy to determine based on firm or market portfolio financial data. Likewise, the model does not assume that dividend growth rates or distribution ratios are fixed over time, which allows us to refine the valuation process. Moreover, the model explicitly integrates and identifies the measure of risk directly into the final valuation formula. Also, the model utilizes a recent and performing

[20] Other empirical difficulties associated with dividend data are also highlighted in Campbell (2000), as Da (2009) mentioned.

[21] In Appendix C, we derive the relationship between a firm's intrinsic value and its accounting cash flow beta using the Taylor series expansion.

[22] For a specific period, the measure of risk corresponds to the accounting consumption beta (obtained from the earnings-consumption covariance). This specific risk measure can be viewed as a particular case of the accounting cash flows beta (the case where $N = 1$).

measure of accounting risk (Da 2009). Besides, it is possible to derive the main results of the model without a specific utility function or a specific probability distribution if we accept the restrictive approximation that results from the first-order Taylor series expansion.

Like many other studies related to the intertemporal CCAPM, we used the time-separable utility assumption. It could be appropriate to generalize the utility function, assuming the presence of habit formation, for example.

Appendix A

In Appendix A, we derive a firm's intrinsic value for one period using the Taylor series expansion, which allows us to replicate the derivation presented in Section 3 without specific utility function or specific probability distribution.

According to Taylor's theorem, we can evaluate the function $y = f(x)$ around the point a in terms of its derivatives in this manner:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + ... + \frac{f^{(N)}(a)(x-a)^N}{N!}.$$

If N is equal to 1, the Taylor series approximation shows that:

$$f(x) \approx f(a) + f'(a)(x-a),$$

and an approximation of the marginal rate of substitution (MRS) can be obtained from the first-order Taylor series, just as Breeden et al. (1989) proposed. Around C_t, we can write:

$$U'(\tilde{C}_{t+1}) \approx U'(C_t) + U''(C_t)(\tilde{C}_{t+1} - C_t). \tag{A1}$$

Knowing that $\tilde{M}_{t+1} \equiv \delta U'(\tilde{C}_{t+1}) / U'(C_t)$, Equation (21) indicates that:

$$E_t[1 + \tilde{g}^a_{i,t+1}] = \frac{1 + R_{F,t+1}}{1 + 1/\mu_{it}} - (1 + R_{F,t+1}) COV_t\left[\frac{\delta U'(\tilde{C}_{t+1})}{U'(C_t)}, \tilde{g}^a_{i,t+1}\right]. \tag{A2}$$

Integrating Equation (A1) into Equation (A2) suggests that the firm's expected abnormal earnings growth approaches the following value:

$$E_t[1+\tilde{g}^a_{i,t+1}] \approx \frac{1+R_{F,t+1}}{1+1/\mu_{it}}$$
$$-(1+R_{F,t+1})COV_t\left[\delta\frac{U'(C_t)+U''(C_t)(\tilde{C}_{t+1}-C_t)}{U'(C_t)}, \tilde{g}^a_{i,t+1}\right]. \quad \text{(A3)}$$

Based on the basic properties of mathematical covariance, after simple manipulations we can write:

$$E_t[1+\tilde{g}^a_{i,t+1}] \approx \frac{1+R_{F,t+1}}{1+1/\mu_{it}} - \frac{\delta U''(C_t)(1+R_{F,t+1})}{U'(C_t)}COV_t[\tilde{C}_{t+1}-C_t, \tilde{g}^a_{i,t+1}]. \quad \text{(A4)}$$

Multiplying by C_t on each side of Equation (A4) indicates that:

$$E_t[1+\tilde{g}^a_{i,t+1}] \approx \frac{1+R_{F,t+1}}{1+1/\mu_{it}} - \frac{\delta U''(C_t)(1+R_{F,t+1})C_t}{U'(C_t)}COV_t\left[\frac{\tilde{C}_{t+1}-C_t}{C_t}, \tilde{g}^a_{i,t+1}\right]. \quad \text{(A5)}$$

Multiplying by $\sigma^2_t[\tilde{g}_{t+1}]$ on each side of Equation (A5) shows that:

$$E_t[1+\tilde{g}^a_{i,t+1}] \approx \frac{1+R_{F,t+1}}{1+1/\mu_{it}}$$
$$-\frac{\delta U''(C_t)(1+R_{F,t+1})C_t\sigma^2_t[\tilde{g}_{t+1}]}{U'(C_t)}\frac{COV_t[\tilde{g}_{t+1}, \tilde{g}^a_{i,t+1}]}{\sigma^2_t[\tilde{g}_{t+1}]}. \quad \text{(A6)}$$

In this manner, we can replicate the preceding relationship between the firm's expected abnormal growth rate and its accounting consumption beta for one period, that is to say:

$$E_t[1+\tilde{g}^a_{i,t+1}] \approx (1+R_{F,t+1})/(1+1/\mu_{it}) + \lambda_t\beta^a_{it}, \quad \text{(A7)}$$

where

$$\lambda_t \equiv \delta\sigma^2_t[\tilde{g}_{t+1}](1+R_{F,t+1})(-1)C_tU''(C_t)/U'(C_t),$$

$$\lambda_t \equiv \delta\sigma^2_t[\tilde{g}_{t+1}](1+R_{F,t+1})RRA,$$

with

$$RRA \equiv -C_t U''(C_t) / U'(C_t) .$$

The above term *RRA* represents the relative risk aversion, evaluated at C_t. The term value is necessarily positive because the second derivative of the utility function must be negative by construction, and the other values in the *RRA* term must be positive. Also, the new parameter lambda is positive, because the variance of a random variable, the risk-free rate of return, and the time discount factor are positive ($\hat{\lambda}_t > 0$). Therefore, from Equation (25) to Equation (30), we have:

$$V_{it} \approx Y_{it} + \frac{E_t[1+\tilde{g}^a_{i,t+1}] - \hat{\lambda}_t \beta^a_{it}}{R_{F,t+1} + \hat{\lambda}_t \beta^a_{it} - E_t[\tilde{g}^a_{i,t+1}]} X^a_{it} , \tag{A8}$$

where the parameter $\hat{\lambda}_t$ is similar to the parameter λ_t derived in Section 3, except that the derivation of the intrinsic value does not refer to a specific utility function or a specific probability distribution. However, in Appendix A, we must accept the approximation that results from the first-order Taylor series.

Appendix B

In Appendix B, we derive the relationship between a firm's intrinsic value and the accounting cash flow beta, using the Taylor series expansion one more time. Using the Taylor series, we can rewrite Equation (41) in this way:

$$\begin{aligned} E_t[1+\tilde{g}^a_{i,t+n+1}] &\approx \frac{1+R_{F,t+n+1}}{1+1/\mu_{i,t+n}} \\ &- (1+R_{F,t+n+1}) COV_t\left[\delta \frac{U'(C_{t+n}) + U''(C_{t+n})(\tilde{C}_{t+n+1} - C_t)}{U'(C_{t+n})}, \tilde{g}^a_{i,t+n+1}\right]. \end{aligned} \tag{B1}$$

From Equation (A3) to Equation (A6), we can write:

$$\begin{aligned} E_t[1+\tilde{g}^a_{i,t+n+1}] &\approx \frac{1+R_{F,t+n+1}}{1+1/\mu_{i,t+n}} \\ &- \frac{\delta U''(C_{t+n})(1+R_{F,t+n+1}) C_{t+n} \sigma^2_t[\tilde{g}_{t+n+1}]}{U'(C_{t+n})} \frac{COV_t[\tilde{g}_{t+n+1}, \tilde{g}^a_{i,t+n+1}]}{\sigma^2_t[\tilde{g}_{t+n+1}]} , \end{aligned} \tag{B2}$$

or, if we prefer:

$$E_t[1+\tilde{g}^a_{i,t+n+1}] \approx (1+R_{F,t+n+1})/(1+1/\mu_{i,t+n}) + \lambda_{t+n}\beta^a_{i,t+n}, \tag{B3}$$

where $\lambda_{t+n} \equiv \delta\sigma_t^2[\tilde{g}_{t+n+1}](1+R_{F,t+n+1})(-1)C_{t+n}U''(C_{t+n})/U'(C_{t+n})$. Summing from n = 0 to n = N-1, and assuming that $\mu_{i,t+n}$ is stationary, we have:

$$\sum_{n=0}^{N-1} E_t[1+\tilde{g}^a_{i,t+n+1}] = (1+1/\mu_{it})^{-1}\sum_{n=0}^{N-1}(1+R_{F,t+n+1}) + \sum_{n=0}^{N-1}\lambda_{t+n}\beta^a_{i,t+n}. \tag{B4}$$

Multiplying by the scalar value $\sum_{n=0}^{N-1}\lambda_{t+n}$ on each side of Equation (B4) shows that:

$$\sum_{n=0}^{N-1} E_t[1+\tilde{g}^a_{i,t+n+1}] = (1+1/\mu_{it})^{-1}\sum_{n=0}^{N-1}(1+R_{F,t+n+1}) + \sum_{n=0}^{N-1}\lambda_{t+n}\sum_{n=0}^{N-1}\omega_{t+n}\beta^a_{i,t+n}, \tag{B5}$$

where $\omega_{t+n} \equiv \lambda_{t+n}/\sum_{n=0}^{N-1}\lambda_{t+n}$, with $0<\omega_{t+n}<1$. Dividing by N on each side of Equation (B5) indicates that:

$$1+\bar{g}^a_{it} = (1+\bar{R}_{Ft})/(1+1/\mu_{it}) + \bar{\lambda}_t\bar{\mathrm{B}}^a_{it}, \tag{B6}$$

where $\bar{\lambda}_t \equiv \sum_{n=0}^{N-1}\lambda_{t+n}/N$, $\bar{\mathrm{B}}^a_{it} \equiv \sum_{n=0}^{N-1}\omega_{t+n}\beta^a_{i,t+n}$. Thus, after simple manipulations, we finally have:

$$V_{it} = Y_{it} + \frac{1+\bar{g}^a_{it} - \bar{\lambda}_t\bar{\mathrm{B}}^a_{it}}{\bar{R}_{Ft} + \bar{\lambda}_t\bar{\mathrm{B}}^a_{it} - \bar{g}^a_{it}}X^a_{it}, \tag{B7}$$

where parameter $\bar{\lambda}_t$ and parameter $\bar{\mathrm{B}}^a_{it}$ are similar to parameters $\bar{\lambda}_t$ and $\bar{\beta}^a_{it}$ respectively, derived in Section 4, except that the derivation of the intrinsic value does not refer to a specific utility function or a specific probability distribution (as in Appendix A).

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