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COMMENT

On the Empirical Status of the Matching Law: Comment on McDowell (2013)

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The matching law, regardless of the version, is a mathematical model that accounts for an organism's response rate as a function of the reinforcer rate. McDowell (2013) investigated to which extent a combined version of the quantitative law of effect (Herrnstein, 1970) and the generalized matching law (Baum, 1974) accounts for a substantial amount of the variance through several data sets. Even if I agree with most points raised by McDowell, there are 2 important issues within his reanalysis. Two out of 6 studies relied on pooled-subject data that are inappropriate for an investigation of the matching law (Caron, 2013). Moreover, the combined equation was not systemically investigated through all data sets. The current study casts some doubt on the empirical status of modern matching equations and thus shows that they still deserve extensive attention.

Keywords: choice, matching law, pooled data, within-subject variance

The matching law is a quantitative model that describes the response allocation of an organism according to the relative reinforcer ratio (Herrnstein, 1961). The model has evolved into two equations: the quantitative law of effect proposed by Herrnstein (1970) and the generalized matching law proposed by Baum (1974). The quantitative law of effect conceptualizes the absolute response rate as a hyperbolic function of the absolute reinforcer rate, respectively, *B*s and *r*s in Equation 1:

$$B = \frac{kr}{r + r_e}. \tag{1}$$

Theoretically, the parameter k corresponds to absolute response rate, and r_e corresponds to extraneous reinforcers. Herrnstein's (1970) conceptualization implies that the absolute quantity of behavior and extraneous reinforcers are constant within an experimental condition. Thus, the quantitative law of effect is more a theory than a purely descriptive equation such as the generalized matching law,

$$\frac{B_1}{B_2} = b\left(\frac{r_1}{r_2}\right)^a, \tag{2}$$

where *B*s and *r*s are the same as Equation 1. The generalized matching law conceptualizes response ratios and reinforcer ratios as a power function. The exponent a is referred to as sensitivity, and the coefficient b is referred to as bias. The power function is also known in its logarithmic form:

$$\log\left(\frac{B_1}{B_2}\right) = a\log\left(\frac{r_1}{r_2}\right) + \log b. \tag{3}$$

Every parameter, a, b, r_e, k, and the explained variance from each equation, are obtained via an ordinary least-squares regression where parameters are generally free to vary, even though fewer studies imposed constraints on the parameters.

McDowell (2013) attempted to unify both equations into a single framework. He evaluated through extensive data sets to which extent the modern matching equations (Equations 6–9 from the target article) can account for a substantial quantity of the variance and whether residuals appeared systematically correlated. However, McDowell did not systematically investigate Equation 6′ from target article and numbered alike here. Moreover, McDowell used two conceptually inappropriate data sets out of six sets. Instead of analyzing the matching law from single-subject data, he conducted analyses on pooled-subject data. Therefore, his analyses violate a simple assumption of matching theory, that is, the matching law describes individual choices.

Unsystematic Analyses

McDowell (2013) investigated to which extent Equation 6′ accounted for variances by imposing constraints on the parameters. When fitting equations to data sets, parameters could be unconstrained, free to vary across conditions, or constrained, share across conditions. The equation is repeated below:

$$B = \frac{kr^a}{r^a + c_{1e}}, \tag{6′}$$

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where

$$c_{1e} = \frac{r_e^a}{b_{1e}}$$

or a parameter combining extraneous reinforcer, sensitivity and bias. However, McDowell did not compare systematically the constraints imposed on the parameters of Equation 6′. Table 1 shows constraints according to the associated study. Note that neither *a* nor *b* was ever shared across conditions, which is theoretically appropriate. However, constraints on parameters *k* and r_e were varied without much specification. Indeed, these two parameters were free to vary across conditions in two analyses (data sets from Dallery, McDowell, & Lancaster, 2000; McDowell & Dallery, 1999): *k* was constrained and r_e was free in one analysis (averaged data from Dallery et al., 2000), and both were constrained in the last analysis (Soto, McDowell, & Dallery, 2005). Still, McDowell could have investigated the theoretical equal-*k* or nonequal-*k* requirements by systematically analyzing every study with all possible constraints on parameters *k* and r_e. Unfortunately, investigations of whether Equation 6′ accounts for more or less variances across studies are unreliable because they are not subject to the same constraints.

Pooled-Subject Data

Recently, Caron (2013) argued that parameters from pooled-subject analyses are unrepresentative of within-subject matching. Statistically, when using pooled-subject data, parameters are fitted according to the between-subjects variance rather than the within-subject variance. However, matching studies are concerned about describing behavior of a single organism or, more specifically, on how it responds to variations in reinforcer rate. Thus, researchers want to explain the within-subject variance.

McDowell's (2013) conclusions, based on McDowell and Caron's (2010a, 2010b) data, are then strongly suspected to inadequately describe individual choice, because they are based on between-subjects variances. McDowell and Caron investigated whether the generalized matching law describes rule break and normative talks of delinquent boys. They found via pooled-subject data that modern matching equations described the relation accurately. Moreover, when boys were divided into quartiles according to their level of deviance, results showed that increasing deviance-level increased undermatching ($a < 1$) and decreased bias in favor of normative talk. However, these results are doubtful because there is no measure of matching at an individual level. In other words, each subject does not have an individual measure of the bias to correlate with his level of deviance. Every subject corresponds to a single data point and, as such, individual variance cannot be accounted by a regression. Finally, how boys' deviance level influenced their own matching parameters remains unknown.

Table 1
Constraints Imposed on the Parameters of Equation 6′ According to the Experiment

Experiment	*k*	r_e	*a*	*b*
McDowell & Dallery (1999)[a]	Free	Free	Free	Free
Dallery et al. (2000)	Constrained	Constrained	Free	Free
Dallery et al. (2000), avg.	Constrained	Free	Free	Free
Soto et al. (2005)	Free	Free	Free	Free

Note. Avg. = averaged.
[a] Reanalyzed by McDowell (2005).

Discussion

The purpose of the current comment was to identify the statistical issue arising from pooled-subject data and unsystematic re-analyses. Nevertheless, I have to agree with most claims raised by McDowell. The strict matching law and the quantitative law of effect (Equations 1–4 in the target article) are certainly false (Davison & McCarthy, 1988; McLean, 2006; Warren-Boulton, Silberberg, Gray, & Ollom, 1985), regardless of McDowell's analyses. Moreover, modern matching equations, such as the generalized matching law (Equation 5 in the target article), are the current trend in the experimental analysis of behavior literature (Cording, McLean, & Grace, 2011; Davison & Baum, 2000; McDowell, 2005; Sutton, Grace, McLean, & Baum, 2008). I also acknowledge the effort to gather the enormous amount of studies on matching in a single coherent article. Despite this apparent agreement, McDowell's data analyses are inappropriate to raise the important conclusion of the target article. Future studies should therefore investigate systematically the viability of modern matching equation to individual-subject data. In conclusion, the empirical status of the matching law still deserves extensive attention.

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