Upscaledb: Efficient Integer-Key Compression in a Key-Value Store using SIMD Instructions

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Abstract

Compression can sometimes improve performance by making more of the data available to the processors faster. We consider the compression of integer keys in a B+-tree index. For this purpose, systems such as IBM DB2 use variable-byte compression over differentially coded keys. We revisit this problem with various compression alternatives such as Google's VarIntGB, Binary Packing and Frame-of-Reference. In all cases, we describe algorithms that can operate directly on compressed data. Many of our alternatives exploit the single-instruction-multiple-data (SIMD) instructions supported by modern CPUs. We evaluate our techniques in a database environment provided by Upscaledb, a production-quality key-value database. Our best techniques are SIMD accelerated: they simultaneously reduce memory usage while improving single-threaded speeds. In particular, a differentially coded SIMD binary-packing techniques (BP128) can offer a superior query speed (e.g., 40 % better than an uncompressed database) while providing the best compression (e.g., by a factor of ten). For analytic workloads, our fast compression techniques offer compelling benefits. Our software is available as open source.

Keywords: B+-tree, Data Compression, Vectorization, Key-Value Stores

1. Introduction

The B-tree and its variations (such as the B+-tree) have been ubiquitous in computing since the 1970s [2]. Almost all relational database engines use data structures resembling the B-tree. Moreover, many applications rely on an embedded database to store key-value pairs in B-trees. There are many popular choices today including Berkeley DB [28], Kyoto Cabinet and Upscaledb.

Compressing B-trees can help reduce storage and it may even accelerate some queries by easing the data transfer bottleneck. Indeed, many operations in modern databases leave the CPU idle, waiting for data to arrive either from RAM or from the disk. Applications where updates are infrequent and queries have low selectivity (i.e., analytic workloads) are especially likely to benefit from compression.

Short fixed-length keys are well suited for advanced optimization. Of particular interest is the case where keys are integer values (e.g., 32-bit integers). They are relevant when the key is an identifier (e.g., IP address, user ID, row ID and so forth).

To get substantial performance benefits out of compression, we must ensure that decompression is fast enough. For this purpose, IBM DB2 uses variable-byte compression and differential coding [3]. In a differential-coding model, instead of storing the integer keys themselves $(x_1, x_2, \ldots)$, we store the successive differences $(x_1 - 0, x_2 - x_1, x_3 - x_2, \ldots)$. These differences are typically small when the values are maintained in sorted order. Small integers can be compressed quickly (see § 2). The compression technique used by IBM DB2 is well established, fast and it provides reasonable compression ratios.

However, there is a wide range of integer compression techniques, and many of them have been modified to fully benefit from modern processors. For example, commodity processors (e.g., ARM, POWER, Intel, AMD) provide instructions operating on wide 128-bit registers (called *XMM registers* on Intel platforms). These wide registers can be used to operate on many values at once (e.g., four 32-bit integers). So we can add or subtract four pairs of 32-bit integers using a sin-

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gle instruction. We qualify these instructions as being single-instruction-multiple-data (SIMD). They are helpful when compressing arrays of integer values [21, 43].

Can optimized compression techniques accelerate a key-value database system? To find out, we implemented a set of state-of-the-art compression techniques in an established open-source embedded database engine (Upscaledb) based on a B+-tree. We also implemented accompanying algorithms over compressed data to support common operations. We provide an experimental comparison. To our knowledge, such a comparison has never been attempted, especially not one that includes SIMD-accelerated schemes.

Unsurprisingly, we show that we can compress the keys of the B+-tree to a tenth of the original size. This ensures that more data can be stored in RAM. Moreover, our results indicate that compression improves real-world performance and that the gains can be substantial (e.g., 40 %). However, we find that only our fastest schemes, those based on SIMD instructions, accelerate B+-tree performance reliably. Our work confirms earlier findings stressing the benefits of designing compression algorithms to leverage SIMD instructions [22, 43].

2. Codecs

We are interested in compressing 32-bit unsigned integers. For most of our compression algorithms, integers are differentially coded prior to compression so that most of them are small. That is, starting from an array of integers $x_1, x_2, \ldots$, we compress the integers $x_1, x_2 - x_1, x_3 - x_2, \ldots$ There are many suitable integer-compression schemes: we selected some likely to offer good performance.

For recent reviews on the topic, see Lemire and Boytsov [21] or Zhao et al. [43].

Differential coding. During decoding, given the differences $\delta_1 = x_1, \delta_2 = x_2 - x_1, \delta_3 = x_3 - x_2, \ldots$, we need to reconstruct $x_1, x_2, x_3, \ldots$ This operation requires the computation of a prefix sum $(\delta_1, \delta_1 + \delta_2, \delta_1 + \delta_2 + \delta_3, \ldots)$. To avoid unnecessary data operations, we integrate the decompression and the prefix sum computation. That is, we output the decoded values all at once: we do not first output the difference $\delta_1, \delta_2, \delta_3, \ldots$ and then the reconstructed decoded values. For SIMD-based schemes, the computation of the prefix sum can be vectorized as follows. We process the data in registers of four 32-bit integers (which call vectors).

1. Shift the vector by two integers $(\delta_i, \delta_{i+1}, \delta_{i+2}, \delta_{i+3}) \to (0, 0, \delta_i, \delta_{i+1})$.
2. Add the original delta vector with the shifted version $(\delta_i, \delta_{i+1}, \delta_i + \delta_{i+2}, \delta_{i+1} + \delta_{i+3})$.
3. Shift the vector by one integer $(\delta_i, \delta_{i+1}, \delta_i + \delta_{i+2}, \delta_{i+1} + \delta_{i+3}) \to (0, \delta_i, \delta_{i+1}, \delta_i + \delta_{i+2})$.
4. Add the previous vector with the shifted version $(\delta_i, \delta_i + \delta_{i+1}, \delta_i + \delta_{i+1} + \delta_{i+2}, \delta_i + \delta_{i+1} + \delta_{i+2} + \delta_{i+3})$.

This vectorized approach can be much more efficient than a naive scalar implementation [22].

Core functions. Beside compression and decompression, all our schemes need to support at least three core functions:

- In a selection, we seek the i^{th} integer. While we can always fully decompress all the integers and seek the result in the uncompressed array, we can typically do much better with a specialized function that avoids unnecessary data manipulation—ideally accessing only the necessary data and avoiding committing to memory intermediate register values.
- We assume that the integers are stored in sorted order, and we want to seek the location of the first value greater or equal to a given target, and to retrieve this value. Again, though we can implement this function by fully decompressing the data, we can often do much better by avoiding a full decompression.
- We also need to implement an in-order insertion. That is, under the assumption that the values are in sorted order, we need to be able to add one more value, while maintaining the result in sorted order.

In all three cases, we are targeting relatively small compressed blocks (e.g., 256 integers). In instances where there are many more integers, a search might rely on auxiliary data structures. In our case, the B+-tree itself provides the indexing so that all the compressed data can be assumed to fit CPU cache.

Delete stability. All our schemes, except for binary packing (BP128), satisfy a property that we call *delete stability*: the removal of a value may not increase the storage requirement. It was called the "Delete Safe Property" by Bhattacharjee et al. [3] in the context of IBM DB2 where it is required as a design principle. Without this property, we may get the possibly unexpected effect that removing a key increases the storage requirement. To see why delete stability should not be taken from granted, consider the list of integers $\{1, 2, 3, 4, \ldots\}$. Their successive differences are

Table 1: VByte compression of various integer values. The most significant bit of each byte is in bold.

integer	VByte
1	**0**0000001
2	**0**0000010
128	**1**0000000, **0**0000001
256	**1**0000000, **0**0000010
32768	**1**0000000, **1**0000000, **0**0000010

$\{1, 1, 1, 1, \ldots\}$. However, suppose that we delete the second value, getting $\{1, 3, 4, \ldots\}$, then the successive differences contain a larger value (2): $\{1, 2, 1, 1, \ldots\}$. Because the differences contain a larger value, they might be less compressible—depending on the compression algorithm used.

2.1. VByte

VByte [4, 6, 34, 37, 41] is one of the most popular and established integer compression techniques. It is also known as variable-byte, var-byte, varint and escaping. We find it in common interchange formats (such as Google's Protocol Buffer) as well as in search engines (such as Apache Lucene). Starting from the least significant bits, we write non-negative integers using seven bits in each byte, with the most significant bit of each byte set to 0 (for the last byte of an integer), or to 1. Integers in $[0, 2^7)$ are coded using a single byte, integers in $[2^7, 2^{14})$ are coded using two bytes and so on. See Table 1 for examples. There are many inconsequential variations on this format, e.g., we could use the value 1 to indicate the last byte of an integer instead of the value 0.

For large 32-bit input values ($> 2^{28}$), VByte is inefficient because it requires more storage (5 bytes) than the uncompressed value (4 bytes). However, such a problem is uncommon if we use differential coding on sorted inputs.

Our VByte implementation uses standard C code. VByte decoding can be fast when most integers fit in a single byte. One can then decode over a billion integers per second on commodity superscalar desktop processors. However the performance can be lower when integers fit in various numbers of bytes, as the cost of branch mispredictions increases. Indeed, the decode function requires frequent CPU branches when checking the continuation bits and there are data dependencies when decoding a value. In the worst case, the CPU needs to process bytes sequentially. Several other schemes were created to overcome this limitation, two of them were included in our tests: Masked VByte [31] and VarIntGB [12].

Implementing a fast select or fast (sequential) search over VByte—without first decompressing all integers—is relatively straightforward. Moreover, VByte can support fast insertions. That is, suppose that we want to insert value a that would first between values x_i and x_{i+1}. There is no need to change any of the bytes corresponding to the values $x_1, \ldots, x_i$. After these bytes, the data corresponding to $x_{i+1} - x_i$ must be updated so that we can store the bytes corresponding to $a - x_i$ and $x_{i+1} - a$. Meanwhile, all the bytes corresponding to the values $x_{i+2}, x_{i+3}, \ldots$ do not need to be modified, so that they can be merely moved in memory. This observation is not novel: Büttcher and Clarke remark that we can often merge two variable-byte stream without having to recompress them [7]. This makes VByte convenient from an engineering point of view.

To summarize, VByte's strength is its simplicity and convenience. It can be implemented in just a few lines of code. Copying and splitting encoded sequences do not require a re-encoding of the data which enables fast operations on compressed values, i.e., to insert or delete values.

2.2. VarIntGB

VarIntGB [12] was engineered by Google to alleviate the performance problems that VByte was causing. The main insight is that instead of encoding and decoding integers one at a time, we can encode and decode blocks of 4 integers instead.

In VarIntGB, we use $\lceil \log_{2^8}(x + 1) \rceil$ continuous bytes to store an integer x. That is, if x is in $[0, 2^8)$, we use one byte, if x is in $[2^8, 2^{16})$, we use two bytes, and so forth. Given four integers, x_1, x_2, x_3, x_4, we first compute the array of byte widths $M = \lceil \log_{2^8}(x_1 + 1) \rceil, \lceil \log_{2^8}(x_2 + 1) \rceil, \lceil \log_{2^8}(x_3 + 1) \rceil, \lceil \log_{2^8}(x_4 + 1) \rceil$. This array is made of four integers in $\{1, 2, 3, 4\}$. We can store each of these values using 2 bits. Thus, the array M can be stored in a single byte (8 bits). The VarIntGB format encodes each block of four integers using one byte for the array M, and then $M_1 + M_2 + M_3 + M_3$ bytes to represent the data corresponding to integers x_1, x_2, x_3, x_4 respectively. See Fig. 1. In practice, if the number of integers is not divisible by four, we may end with a partial block.

When decoding VarIntGB data, we first examine the byte M, and extract its four components. The integers x_1, x_2, x_3, x_4 are then decoded from the following $M_1 + M_2 + M_3 + M_3$ bytes. Unlike VByte, the decoding of VarIntGB requires a fixed number of operations

Figure 1: Encoded bytes corresponding to integer values 1024, 12, 10, 512 using VarIntGB. The result occupies seven bytes.

01—00—00—01	0x0004	0xc	0xa	0x0200

per encoded integer in our implementation. Thus, VarIntGB can be expected to offer a better performance than VByte over poorly compressible data. Moreover, we can address the important case where all four integers are stored using a single byte, and decode the four integers quickly, thus achieving a superior performance for highly compressible data.

Like VByte, implementing a fast select or a fast sequential search over VarIntGB is straightforward. However, a fast insertion is more difficult. If we find that the values must be inserted at index i, then we can certainly avoid rewriting the first $\lceil i/4 \rceil$ values. However, recoding remaining values in-place following the insertion of a single new value is both technically cumbersome and relatively slow. We found it more appropriate to decompress the remaining values, and then recompress them with the new value added.

2.3. Masked VByte

Our VByte decoder algorithmically processes one input byte at a time. Though VarIntGB accelerates the processing by changing the format, it might also be possible to accelerate VByte itself by designing a SIMD-based decoder. Indeed, the VByte format might be otherwise convenient, if only because it is a well-known and time-tested format. Stepanov et al. [34] tested such a SIMD-based decoder but got only modest gains (less than 25 %). However, Plaisance et al. [31] showed for the first time that it is possible to multiply VByte decoding speed by using SIMD instructions in a decoder called Masked VByte.

The Masked VByte approach first gathers the most significant bits of an array of consecutive bytes. A single instruction (`pmovmskb`) can serve for this purpose on Intel processors. To illustrate the approach, suppose we code the integers 128, 386, 16, 32 using VByte. The result will be six compressed bytes: **1**0000000, **0**0000001, **1**0000010, **0**0000011, **0**0010000, and **0**0100000. We can gather the control bits (1,0,1,0,0,0 in this case) using the `pmovmskb` instruction. We can then use another instruction (`pshufb`) that permutes the bytes of a register in a desired way (that may be specified using a control mask) in as little as one cycle. Finally, we need to mask or shift the bytes to arrive at the decoded integers. The exact algorithm and its implementation is technical and requires many steps, so we refer the reader to the original work for details [31].

The Masked VByte can be three times faster than a conventional VByte decoder when the data is sufficiently compressible. Masked VByte also benefits from an integrated SIMD-accelerated differential coding.

We implemented accelerated select and sequential search functions that are similar to the VByte functions. Selection is SIMD-accelerated: we decode in registers the first $4\lceil i/4 \rceil$ values and return the value at the proper index. Sequential search is handled similarly. For the insertion, we use the same function as for VByte: this is possible because the underlying data format is identical.

2.4. Binary Packing (BP128)

The binary representation of an unsigned integer x has all but the less significant $\lceil \log_2(x+1) \rceil$ bits equal to zero. Given an array of integers $x_1, x_2, \ldots$, if m is the maximum value ($m = \max x_1, x_2, \ldots$), then we can store all integers using $\lceil \log_2(m+1) \rceil$ bits per integer. Effectively, we can truncate the leading zeros. Binary packing is a compression technique that exploits this idea. We regroup all integers in blocks of, say, 128 integers. We find $b = \lceil \log_2(m+1) \rceil$ and store it using as little as a byte, and then we write out the 128 integers using b bits per integer, packing them tightly so that $128b$ bits are used in total.

Like with VarIntGB, if the number of integers is not divisible by 128 integers, we may end with a partial block. For simplicity, we may pad the input with zeros so that the number of integers is divisible by 128.

Lemire et al. [22] describe an optimized SIMD-accelerated version of binary packing, henceforth BP128, where blocks of 128 integers are used. Differential coding is integrated in the unpacking routines.

We implemented fast functions to select just a single value, that is, if the i^{th} value is sought, only the first $4\lceil i/4 \rceil$ values are decoded (in registers) and the desired value is returned.

We implemented a similar sequential search function. Insertions require decoding the entire array, editing the uncompressed data and recompressing.

2.5. Frame of Reference

Though all compression techniques presented so far rely on differential coding, there are other alternatives. Indeed, differential coding has the downside that it is relatively difficult to random skip values: without auxiliary data structures, one must decode integers starting from the beginning. In particular, it might be difficult to use a fast search technique, like binary search, directly on the compressed data.

One convenient compression alternative that offers good compression as well as fast random access is Frame-Of-Reference (FOR) [13]. In FOR, arrays of values are partitioned into blocks (e.g., of 32 integers). We code the minimal value of the block, and then all values are written in reference to this minimum. For example, the block of values $\{500, 521, 531, 574\}$ would be written as $\{21, 31, 74\}$. To decode these values, all we need is the minimum (500), and then we can compute the sums $\{500, 21 + 500, 31 + 500, 74 + 500\}$. When the arrays are sorted, as in our application, the minimal value is always the first one. Thus, starting from the array of sorted values $x_1, x_2, x_3, \ldots, x_n$, we pack the array $x_1 - x_1, x_2 - x_1, x_3 - x_1, \ldots, x_n - x_1$ of transformed integers using $\lceil \log_2(x_n - x_1 + 1) \rceil$ bits per integer, packing and unpacking them quickly, as in binary packing. In our implementation, we use blocks that are multiples of 32 integers for the scalar version (henceforth FOR) and multiple of 128 integers for the SIMD-accelerated version (henceforth SIMD FOR). When the number of input integers is not divisible by the block size, we create a partial block, packing only the necessary x integers, to maximize the compression ratio.

One can select the i^{th} value in a block in constant time (irrespective of the block size). If the values are sorted, searching for a value can be done in logarithmic time using binary search. Inserting a new value can be done by uncompressing the block, inserting the value in the uncompressed data and then recompressing it.

3. Database Compression in Upscaledb

Upscaledb is an embedded key-value database engine implemented in C++. It is used for a variety of tasks like caching web crawler data, gathering and pre-processing sensor input and network events, storing the metadata of backup software or just as a general data store for mobile and desktop applications.

Upscaledb's functionality is similar to other key-value database engines like Berkeley DB, but different in its implementation. Instead of using type-less byte arrays as keys, Upscaledb is aware of the key type. It can therefore optimize the underlying data structures and algorithms for the type.

The key type is specified by the user when creating a database. Among the supported types are 32-bit integers, 64-bit integers, variable-length binary data and fixed-length binary data. This configuration parametrizes C++ template classes for lower-level structures and algorithms.

Upscaledb uses a B+-tree [10] structure to store the database indices. A B+-tree is similar to a conventional B-tree. It is an associative map that stores keys and values in sorted order and supports logarithmic-time search, insertion and deletion operations. However, the B+-tree stores values aligned with the leaf nodes. Moreover, the leaf nodes form a linked list for fast traversal.

Figure 2: Memory layout of a B+-tree node

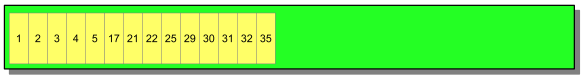


Figure 3: The `KeyList` storing a sorted, non-consecutive sequence of fixed-length integer keys

B+-trees are a common data structure. Indeed, B+-trees are also used by other key-value stores like Berkeley DB and Tokyo Cabinet, as well as many other database engines such as Oracle and SAP HANA.

Upscaledb's B+-tree node (also called a *page*) stores keys and values separately from each other. The actual in-memory layout is described in Fig. 2. Each node has a header structure of 32 bytes containing flags, a key counter, pointers to the left and right siblings and to the child node. This header is followed by the `KeyList` (where we store the key data) and the `RecordList` (where we store the value's data). Their layout depends on the index configuration and data types.

The `RecordList` of an internal node stores 64-bit pointers to child nodes, whereas the `RecordList` of a leaf node stores values or 64-bit pointers to external blobs if the values are too large.

Fixed-length keys (Fig. 3) are always stored sequentially and without overhead. They are implemented as plain C arrays of the key type, i.e., `uint32_t keys[]` for a database of 32-bit integers. Variable-length keys (Fig. 4) use a small in-node index to manage the keys. Long keys are stored in separate blobs; the B+-tree node then points to this blob.

The Upscaledb in-memory representation of fixed-length keys eases the application of compression. The keys are already stored in sorted order, therefore applying differential encoding does not require a change in the memory layout. Since keys are stored sequentially in memory, SIMD instructions can be used efficiently if needed.

3.1. Insertion and Deletion

In a conventional B+-tree, non-root, non-leaf nodes can accommodate between b and $2b$ keys. Leaf nodes

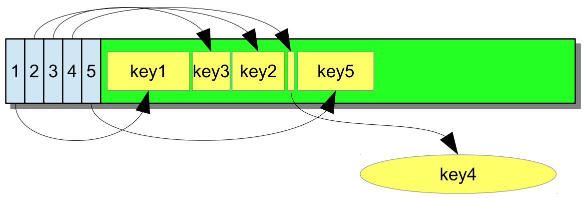


Figure 4: The KeyList storing variable-length keys

accommodate between b and $2b-1$ keys. Nodes are split or merged following insertions and deletions to remain in these ranges. (See Appendix A for details.)

Upscaledb differs from a conventional B+-tree in two significant ways. Firstly, the capacity of nodes is defined in terms of storage space, and not as a number of keys. Secondly, the nodes are balanced locally: the result of an insertion or deletion does not immediately propagate back to the root of the tree. We review these two differences in more details in the rest of this section.

Capacity as storage space Instead of defining capacity as the number of keys that a node may include, Upscaledb fixes the maximal space usage of a node (16 kB by default). This space is used to store both keys and values. Unlike conventional B+-trees that forbid nodes from being less than half full, Upscaledb only considers merging nodes that are nearly empty, i.e., nodes that have less than 4 keys.

Leaf nodes use compression, and the maximal number of keys that can be stored depends on the compressibility of the data. The size of the value entries has also an impact, since they both share the same space.

When inserting a new key in a node with compressed keys, it might be necessary to attempt the insertion to determine how many bytes the new node would use. Only then might we determine that the node is full and needs to be split.

Upscaledb follows this approach. If a key cannot be stored in the current space allocated to the `KeyList` then it tries to reorganize the node (e.g., by growing the `KeyList` at the expense of the `RecordList`, if possible). As a last resort, the node is split and the insert operation is done in one of the new nodes.

As mentioned in § 2, the binary packing schemes violate the principle of "delete stability". A BP128-encoded block can grow if an integer is deleted from the block. This can lead to cases where deleting a key from a B+-tree leaf causes an overflow and triggers a node split. To our knowledge, Upscaledb is unique among B+-tree implementations in that it can split nodes when deleting keys. Such an ability allows us to use BP128 as a codec.

In a B+-tree, most nodes are leaf nodes: there would be little storage gain in compressing non-leaf nodes. For example, to store 20 million keys, we might use 4 non-leaf nodes and 2500 leaf nodes. Consequently, in Upscaledb, only leaf nodes use compression. Thus, deleting a key from a non-leaf node cannot lead to this node being split. IBM DB2 also compresses only leaf nodes [3], for the same reason.

Local balancing When inserting a new key, conventional B+-tree implementations usually first go down the tree to find the right node, and then potentially split the nodes from the bottom up. Moreover, when deleting a key, underfilled nodes are balanced. Splits and balancing operations can require modifications in the B+-tree which are propagated all the way up to the root level.

Upscaledb's implementation is different as it is balancing is only performed locally. It follows Guibas and Sedgewick [15] and examines nodes while descending the tree from the root. Nodes that cannot accommodate a new key are split and underfilled nodes are merged—without propagating changes above the parent. This local balancing may improve query latency.

This localized approach works because any parent node is guaranteed to have space for one more key—irrespective of the value of the new key. Hence, when we split the current node, there is no need to immediately split the parent to make room for one more key. There are two minor inconveniences to this localized approach: it can happen that nodes are split although the split is not immediately required. The other inconvenience is that empty nodes are not pruned if their removal would cause global updates in the B+-tree, since we want to restrict our updates to a local scope only. However, this waste only happens in rare situations, and these empty nodes represent only a small fraction of the total storage.

When descending the B+-tree toward the leaf node, the insert and delete algorithms are identical in their implementation. Only when the leaf is reached, do the algorithms diverge and perform their specific action to either insert or delete a key.

If the operation aborts because a node split is required then the node is split, the parent node and the siblings are updated—without propagating the changes further. This is the case even for deletion operations, where some compression codecs might actually require more space when keys are deleted.

3.2. Integrating Compression

Upscaledb implements the typical operations of key-value databases like inserting or overwriting keys, deleting keys and looking up keys. In addition, bi-directional cursors can iterate over the keys. When there is no compression, many of these operations follow the same pattern: binary search is used to find the position of the (new) key in the node; then the operation is executed at the specified position. This works well because we have very fast random access over uncompressed arrays of keys. With integer compression, more care is needed.

To improve performance, the compressed integers are split in blocks. Therefore, we have the following hierarchy: each leaf node contains a `KeyList` which may contain several blocks. The block size depends on the codec. BP128 stores up to 128 integers per block, all other codecs store up to 256 integers by default. We arrived at these block sizes through empirical evaluation (see § 4.3.2).

Each block is described by a small index structure at the beginning of the `KeyList`, containing the offset of the block in bytes relative to the beginning of the `KeyList`, the number of keys in the block, and the size of the block in bytes (or, in case of BP128, the number of bits required for encoding).

Also, each block stores the start value of the encoded integers. This value serves as the starting value when decoding the differentially coded values in the block and it is used to locate the block of a given key. Blocks can grow, but as soon as they reach a limit they are split.

Blocks are stored sequentially within the `KeyList`. Following insertions and deletions, blocks can contain various numbers of keys. It is even possible for a block to become empty (to become a *gap*). However, the space used by a block is not necessarily reclaimed eagerly as its content is reduced.

If the B+-tree node overflows, it is split. Since this is an expensive operation, several attempts are made to optimize the node's layout and save space, to delay the actual split. Blocks are reorganized and gaps are removed. Also, the space which is assigned to the `RecordList` can be reduced, and assigned to the `KeyList`, and vice versa.

From the following list, each integer codec has to support the `compress` and `decompress` functions. The other functions are not mandatory.

- Compress — Compresses a block of integers to a provided memory location.

  ```
  uint32_t compress_block(Index *
      index, const uint32_t *in,
      uint32_t *out);
  ```

  The `compress` function returns the number of bytes required to compress the data.

- Decompress — Decompresses a block of compressed integers to a buffer.

  ```
  void decompress_block(Index *index,
       const uint32_t *in, uint32_t *
      out);
  ```

- Insert — Inserts a new key in a compressed block. Except for BP128, FOR and SIMD FOR, all codecs provide such a custom insert function: see § 3.3 for details. BP128, FOR and SIMD FOR decode the block, modify the decompressed data and re-encode the block.

  ```
  bool insert(Index *index, uint32_t
      *in, uint32_t key, uint32_t *
      pslot);
  ```

  The insert functions returns false if the key already exists. It also returns the position of the new key in the `KeyList`. This position (the "slot") is then required to insert the corresponding value into the `RecordList`. It is assumed that the block has enough free space to insert another key. Growing or splitting the block is handled by the caller.

  If possible, the insert function implements a fast code path to append a new key at the end. See § 3.4 for details.

- Find — Performs a lower bound find for a key, returns the location of the first value at least as large as the specified key value. This function is used to search for a key.

  ```
  int find_lower_bound(Index *index,
      const uint32_t *in, uint32_t key
      , uint32_t *presult);
  ```

- Delete — Deletes a key. Only implemented by the VByte and Masked VByte codecs. Other codecs decode the block, modify the decompressed data and re-encode the block.

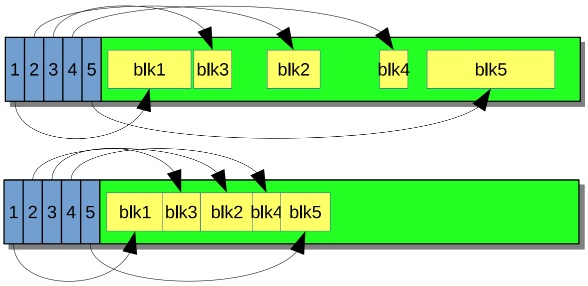


Figure 5: A `KeyList` before (top) and after (bottom) the vacuumize-operation. Fragmentation occurs when empty blocks are deleted. Blocks will have unused space if keys are deleted from a block.

```
template<typename GrowHandler>
void del(Index *index, uint32_t *in
    , int slot,
            GrowHandler *handler);
```

The codecs do not necessarily have "delete stability" (see § 2) and can require more space after a key is deleted. The `GrowHandler` template parameter is used to signal such circumstances to the caller. It can assign more space to the current block or request a B+-tree node split.

- Vacuumize — Reorganizes all blocks, trying to reduce gaps and space to avoid B+-tree node splits.

```
void vacuumize();
```

When "vacuumizing" a `KeyList` (see Fig. 5), the BP128, FOR and SIMD FOR codecs decode all blocks into temporary memory and re-encodes them into new (usually fewer, densely packed) blocks. The other codecs just move the blocks to remove any gaps between the blocks.

3.3. *Fast In-place Updates*

We can shift an array of bytes in memory by a byte offset at high speed. The C language offers the `memmove` function for this purpose, and it is highly optimized—to the point of being limited by the memory throughput. Thus, it is efficient to modify byte-oriented formats in-place (e.g., VByte and VarIntGB). However, we are not aware of any similarly fast function (within a factor of four) to shift an array of bytes by a bit offset that is not divisible by eight. Thus, inserting a value that occupies a number of bits non-divisible by eight in a packed array is likely to be a relatively slow operation. This makes BP128, FOR and SIMD FOR data streams more difficult to update in-place.

In our implementation, VByte, VarIntGB, and Masked VByte perform all update operations directly on compressed data. BP128, FOR and SIMD FOR still require a decode-modify-encode loop for updates.

3.4. *Fast Append Functions*

A common database index operation is to insert new keys at the end, i.e., for time-series data where the key is a chronologically incremented timestamp. Appending integers to an array of differentially-coded compressed integers could be slow if we had to first decode the last value (by summing up the differences). To improve performance, the block descriptor stores the value of the last integer. With this cached value, it is trivial to decide whether a new key is appended at the end or inserted in the middle. If it is appended, then its delta value can be calculated by subtracting the (previously) highest block value from the new value. This optimization improved performance by up to about 30 % for the insertion of sequentially ordered keys. Adding 32 extra bits to the descriptor does not degrade the compression ratios by a significant degree when dozens or even hundreds of keys are stored in a block.

All codecs support fast append functions that avoid first uncompressing the data at least some of the time. For BP128, we only uncompress the block if the existing bit width is insufficient compared to the size of the new delta to be appended. Otherwise, we modify the compressed data directly. We proceed with FOR and SIMD FOR similarly: if the existing bit width is sufficient, we append directly in the compressed data, otherwise we are forced to first uncompress the block.

4. Benchmarks

We first benchmark separately the various operations that are relevant to a key-value store: decompression speed, insertions, select and search. After benchmarking the operations separately, we then report on the performance in Upscaledb with realistic data.

4.1. *Hardware*

All our experiments are executed on an Intel Core i7-4770 CPU (Haswell) with 32 GB memory (DDR3-1600 with double-channel). The CPU has 4 cores of 3.40 GHz each, and 8 MB of L3 cache. Turbo Boost is disabled on the test machine, and the processor is set to run at its highest clock speed. The computer runs Linux Ubuntu 14.04. We report wall-clock time.

Except for the fact that dirty B+-tree nodes in Upscaledb are purged in the background, all tests are single-threaded. Our workloads are small enough to fit in memory and input-output is not a limiting factor.

4.2. Microbenchmarks and Evaluation

We compare the codecs—without database interaction. We compile our C++ benchmarking software using the GNU GCC 4.8.2 compiler with the `-O3` flag. Our implementation is freely available under an open-source license.[1]

Given a bit width $b \leq 24$, we first generate an array of 256 integers in $[0, 2^b)$: $\delta_1, \delta_2, \ldots$. The prefix sum is computed $(\delta_1, \delta_1 + \delta_2, \ldots)$ and used as input data. The result is a sorted list of 32-bit integers. Fig. 6 shows the average compressed size (in bits per integer) and the decompression speed in billions of integers per second. We see that BP128 offers the best compression whereas FOR and SIMD FOR offer poorer compression compared to other codecs.

Regarding the decompression speed, we report the numbers in billions of 32-bit integers decompressed per second (Bis). See in Fig. 6b. SIMD FOR is twice as fast as the next scheme, BP128, which is itself much faster than most other alternatives (up to twice as fast). VByte is the only codec that is limited by a best speed of only about 1 billion integers per second. In contrast, SIMD FOR can decompress data at a rate of over 7 billions integers per second—or about two integers decoded per clock cycle.

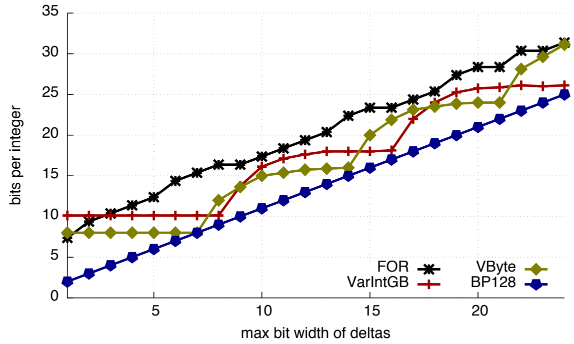
Fig. 7 presents various benchmarks regarding operations on compressed data.

- Instead of starting from the integers in sorted order and compressing them, we pick the integers at random one by one and we insert them in the VByte or VarIntGB stream (Fig. 7a). In the naive implementation, the stream is first decompressed, we insert the value and then recompress the stream. In the fast version, we use our optimized function. We see that the optimized function can be several times faster. We also see that VByte is slightly faster than VarIntGB in this case due to its simpler data layout.

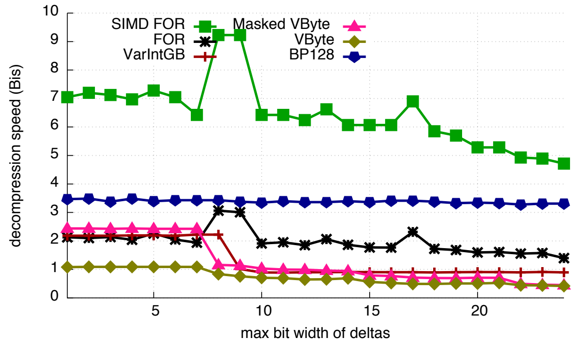
- In Fig. 7b, we randomly select the value at one of the indexes. We present the data in millions of operations per second with a logarithmic scale. We see that FOR and SIMD FOR are an order of magnitude faster at this task because they do not rely on differential coding. BP128 is the next fastest codec while VByte is the slowest.

- In Fig. 7c, we benchmark the find function by randomly seeking a value in range. In this instance, all schemes but VByte are nearly as fast (within a factor of two) for compressible data, while VByte is significantly slower. VarIntGB offers the best performance in this case. FOR and SIMD-FOR differ from the other schemes in that they use a binary search (as a sequential search proved slower) whereas all other codecs rely on a sequential search. If the block size was much larger, FOR and SIMD FOR could be expected to perform better, but we are not interested in that case.

[1] https://github.com/lemire/SIMDCompressionAndIntersection



(a) Bits per integer



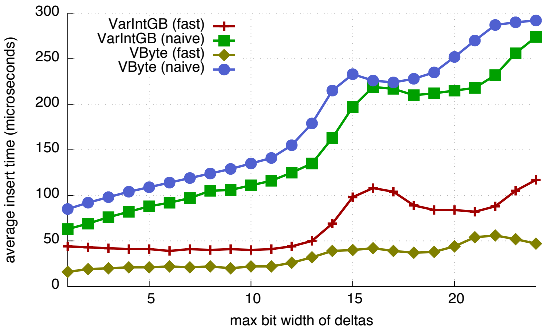
(b) Decompression speed (Bis)

Figure 6: Compression rates and decompression speed

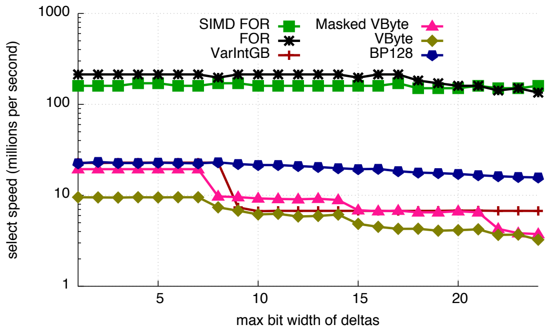
Overall, our results suggest that on speed and compression ratios, BP128 offers good performance. If faster random access is necessary, and the compression ratio is not an issue, then FOR and SIMD-FOR might be preferable.

4.3. In-database Benchmarks

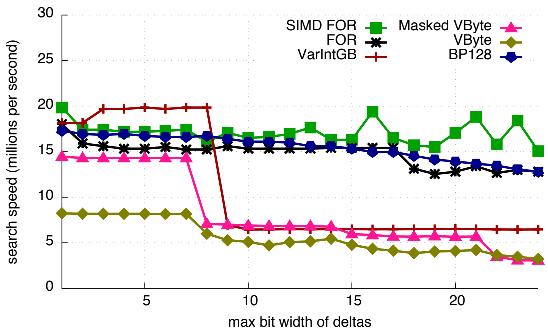
Though synthetic benchmarks show that some compression schemes are superior than others, we are interested in the effect of compression in an actual key-value store. For this purpose, we compiled Upscaledb and our benchmarking software using the GNU GCC 4.8.2 compiler with the `-O3` flag. Upscaledb is freely available



(a) Time to create 256-integer array from random inserts



(b) Select speed



(c) Find speed

Figure 7: Operation timings and speeds over compressed data, for various codecs.

under an open-source license (`http://upscaledb.com`). The benchmarks are executed with "`ups_bench`", a benchmarking tool which is part of Upscaledb's sources.

We aim to study the compression of the integer keys. For this purpose, we only stored the keys, without any accompanying values. All our queries bear only on the keys.

For our experiments, we use the ClusterData model from Anh and Moffat [1]. We vary the number of keys generated, up to a billion. When generating N keys, we set the range of possible values to $[0, 9N/8)$. We insert the keys in order. We choose this data distribution because it is a reasonable model for realistic data.

Table 2 and Fig. 8 present compression results with various database sizes, using the default block sizes (128 for BP128 and 256 for other codecs, see § 4.3.2). Expectedly, the best compression is offered by BP128 which can compress the database by a factor of ten compared to an uncompressed B+-tree. The compression ratios offered by the other codecs are similar (compression ratio of 2 or 3), with SIMD FOR compressing slightly less and VByte compressing slightly better. Both VByte

Table 2: Database sizes in bytes per key, for ClusterData model (N = 20 000 000)

key format	bytes per key
uncompressed	4.02
VByte/Masked VByte	1.06
VarIntGB	1.31
FOR	1.26
SIMD FOR	1.28
BP128	**0.37**

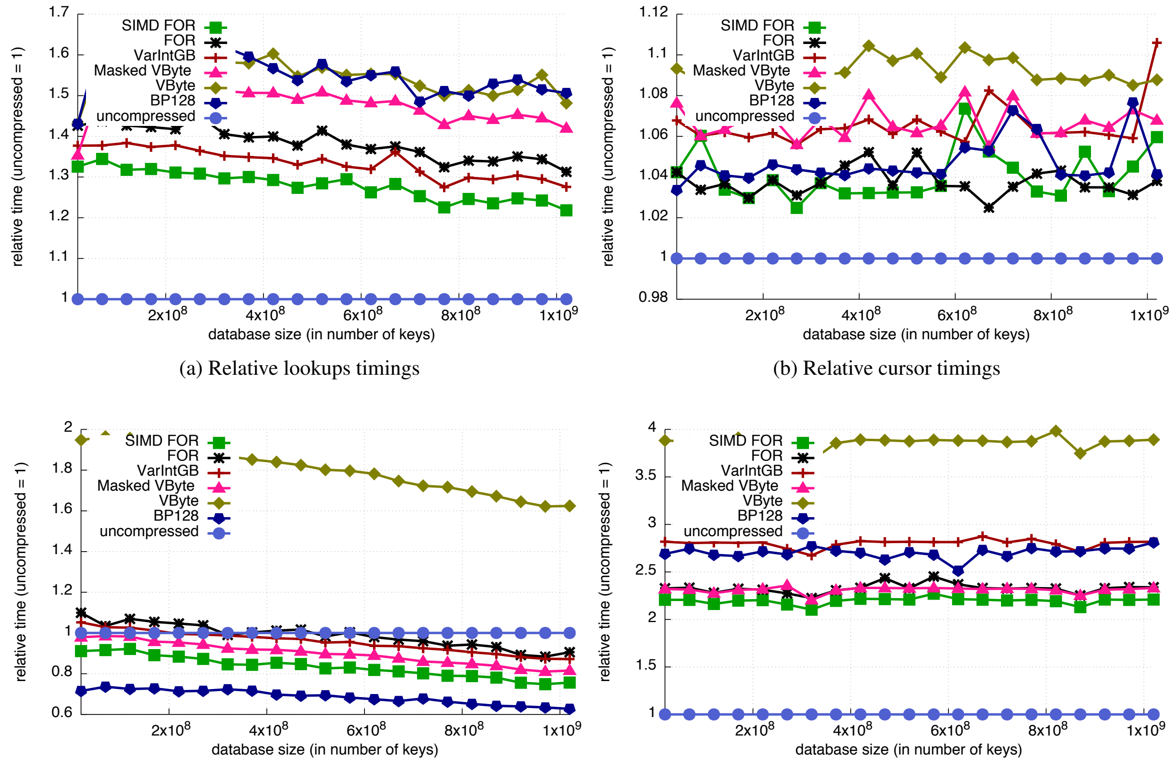
and Masked VByte have exactly the same compressed output. In these tests, we see that the compression as measured by the number of bytes used per key is nearly constant irrespective of the database size.

4.3.1. In-database timings

Each benchmark runs three times, the median result is reported. The difference between the median and other timings is small (typically less than 1 %).

We execute four operations:

Look-up This benchmark opens an existing database



(a) Relative lookups timings

(b) Relative cursor timings

(c) Relative SUM timings

(d) Relative insert timings

Figure 9: Relative timings in Upscaledb (ClusterData).

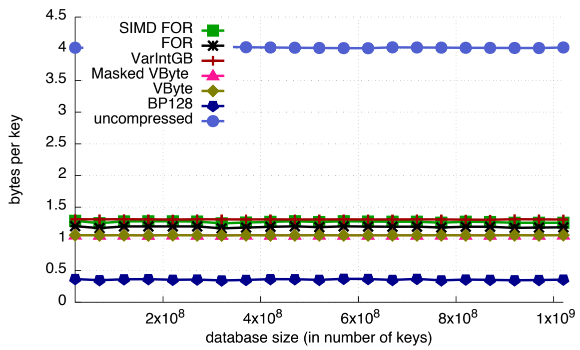


Figure 8: Relative database sizes in Upscaledb (ClusterData)

and performs point-lookups of all inserted keys. Each lookup requires a B+-tree traversal to the leaf node. The node then performs a linear search through the block index and locates the block which stores the requested key. The codec then searches the block for the key.

We implemented search functions for all codecs directly on compressed data. FOR and SIMD FOR do not use differential compression and therefore perform a binary search directly on the compressed data. All other codecs use linear search because they need to rebuild the original value during the search.

The benchmarks show that integer compression does not cause a significant performance hit for lookup operations. Indeed, Fig. 9a shows that the penalty for using compressed keys is about 50 %. We get the best results with SIMD FOR, VarIntGB and FOR (a penalty ranging from 20 % to 40 %) and the worst results with VByte and BP128 (with a penalty of up to 60 %). The good results from VarIntGB in this case are consistent with our microbenchmarks (see § 4.2 and Fig. 7c). BP128 is slightly penalized in this case because it is able to store more keys per leaf node due to its better compression: searching in a node containing more keys takes longer on average, everything else being equal. Though VByte and Masked VByte have the same underlying format, Masked VByte is noticeably faster though not as fast as FOR and SIMD FOR.

Cursor This benchmark opens an existing database and creates a cursor to traverse from the first to the last key. To position itself on the first key, the cursor traverses down the B+-tree at the left-most path down to the leaf, then visits each leaf. Since all leaf nodes are linked, no further traversal is required.

The cursor attaches itself to a leaf node, and stores the current position in the leaf. When the cursor is moved to the next key, this position is incremented. If the last key in the leaf is reached, the cursor loads the sibling of the leaf, attaches itself to the sibling and resets its position to 0.

The cursor then retrieves the key at its current position. If the `KeyList` is uncompressed then the key is accessed with $O(1)$. A compressed `KeyList` first traverses the list of block descriptors, accumulating each block's number of keys till it finds the block which contains the requested key. In our original implementation, the cursor then used a `select` method to retrieve the key directly from the compressed block. But since cursors are usually used for sequential access, and therefore frequently access the same block, we decided to decode the block and cache the decoded values. This causes additional latency when a block is accessed initially, but all following accesses can be served with high throughput. Indeed, our tests showed a significant performance improvement compared to the previous implementation based on `select`.

The final results are presented in Fig. 9b. Compared to an uncompressed database, all codecs except VByte show a penalty of less than about 8 %. VByte does slightly worse with a penalty sometimes exceeding slightly 11 %. Again, though they use the same underlying format, Masked VByte is noticeably faster than VByte.

SUM This benchmark performs a "SUM" operation on all keys. It is equivalent to a `SELECT SUM(column)` operation of a SQL database, where the specified column is an index of unique 32-bit integer keys. For such operations, Upscaledb does not use a cursor to traverse the B+-tree, but performs the operation directly on the B+-tree's data, without copying the keys into the application's memory.

If compression is disabled, the `KeyList` stores all keys in an array of type `uint32_t[]`. The "SUM" operation sums all keys in that array. If compression is enabled, the `KeyList` traverses each compressed block, uncompresses it into temporary memory (in L1 cache) and sums all keys of that memory.

The benchmark results in Fig. 9c show the SIMD accelerated BP128 and SIMD FOR as the clear winners. The compressed databases are even faster than an uncompressed database—with gains reaching 40 % for BP128.

SUM performance is impacted by database size: the bigger the database, the more compression is beneficial, with BP128 and SIMD FOR offering the best performance. Only Masked VByte, BP128 and SIMD FOR are superior to the uncompressed database on the entire test range. VarIntGB and FOR also help the speed for large databases while VByte fails to catch up to the uncompressed database in the scope of our test.

We take this query as a representative of analytic queries where much of the data must be accessed (the query has low selectivity). In such cases, we expect compression to be particularly useful as it reduces data access costs. Upscaledb supports several such queries such as COUNT, COUNT DISTINCT, COUNT_IF, COUNT_DISTINCT_IF, AVERAGE, . . .

Fig. 10 illustrates our results with a more advanced query ("AVERAGE(key) WHERE key > MAX(keys) / 2"). It shows that such queries can be accelerated by the fast compression offered by SIMD FOR and BP128.

Insert This benchmark creates a new database for 32-bit integer keys and inserts various numbers of keys. We should expect a compressed database to be slower for such applications, as insertions may require complete recompression of some blocks—in the worst case.

Fig. 9d shows that among the compressed formats, the best insertion performance is offered by the FOR, SIMD FOR and Masked VByte codecs, followed by BP128 and VarIntGB. VByte is slower than all other codecs. If one uses FOR, SIMD FOR

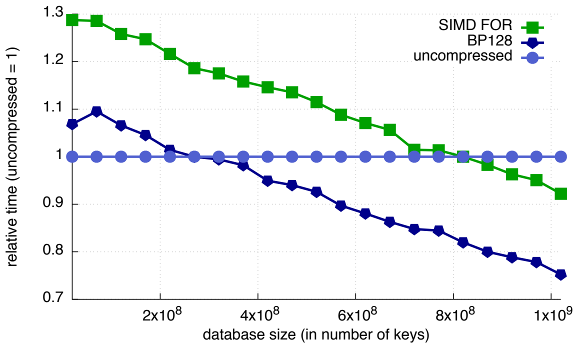


Figure 10: Relative timings in Upscaledb for the query “AVERAGE(key) WHERE key > MAX(keys) / 2” over the ClusterData dataset.

and Masked VByte, insertions in a compressed database are only 2.5× slower than insertions in an uncompressed database.

4.3.2. Setting the block size

Fig. 11 presents the same results for two possible block sizes (128, 256). We experimented with a wide range of block sizes, but only report on these two choices for simplicity. We see that the performance with a larger block size (256) is slightly better and the overall size smaller. The FOR and SIMD FOR codecs benefit substantially from larger block sizes (compared to other codecs) because they rely on a binary search to locate values in the compressed stream: the benefits of a binary search versus a sequential search grow with the size of the blocks. This justifies our design choice of opting for large block sizes (256) for all but one codec. Exceptionally, for BP128, we prefer the smaller block size (128).

5. Related Work

There has been much research dedicated to improving the performance of tree data structures. For example, cache-conscious trees can reduce the number of cache misses to improve performance [20, 24]. In particular, Lee et al. [20] propose the Cache Sensitive T-Trees (CST-Trees).

The application of SIMD instructions to accelerate B+-tree operations (without compression) is reviewed by Zhou and Ross [44]. Willhalm et al. [40] describe how to scan quickly column stores using SIMD instructions. Schlegel et al. [33] show how to accelerate K-ary search on modern processors. Raman et al. [32] describe the IBM DB2 column store that makes extensive use of SIMD instructions and compression.

Compression in databases has a long history [39]. Compression techniques such as run-length encoding and differential coding are common, for example, in column-oriented databases [35]. IBM DB2 compresses integer keys using variable-byte compression and differential coding [3]. Graefe [14] describes the compression opportunity when keys are consecutive.

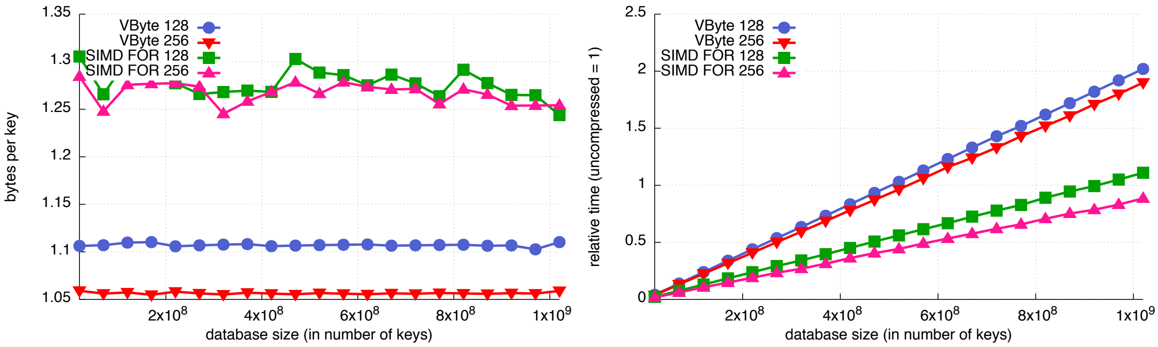
Jin and Chung improve the CST-Trees by using FOR compression [16]. Similarly, Kim et al. [18] propose a SIMD-accelerated in-memory tree index (FAST) where they use FOR. In related work, Yamamuro et al. [42] propose the VAST-Tree: it improves upon FAST in several ways. In particular, it offers better compression ratios of the keys than FAST by using differential coding and the PFOR compression scheme [45]. Though fast, PFOR does not exploit SIMD instructions: Lemire and Boytsov [21] found that SIMD-accelerated binary packing (i.e., BP128) could be 2 to 3 times faster with little difference in the compression ratios.

Random access in differentially-coded compressed arrays is often made possible with auxiliary data structures that allow skipping [26, 37]. However, there are alternatives to differential coding that offer more convenient random access. Claude et al. [9] propose differentially encoded search trees; Teuhola [36] adapts interpolative coding [25] so that it can support logarithmic search. Brisaboa et al. [5] modify variable-byte encoding to create Directly Addressable Codes (DACs)—so that one can have access to individual coded value in constant time using rank/select dictionaries. This strategy is applied to other compression schemes by Külekci [19]. There has also been much interest in variations on the Elias-Fano representation [30, 38], as it can provide good compression and fast random access to the encoded values. Other techniques such as wavelet trees [27] or bitmap indexes [11, 17, 22, 23] can also be used for similar purposes.

Our work should be applicable to other B-trees and related data structures, i.e., Log-Structured Merge-Trees [8, 29] (LSM).

6. Conclusion

We have shown that fast key compression could improve the single-threaded performance of a key-value store—if these compression techniques are accelerated with SIMD instructions. One of our best performing codec (BP128) has the property that the removal of a key may (slightly) increase the storage requirement: something that the IBM DB2 design team specifically excluded. We have presented a practical B+-tree imple-



(a) Database sizes

(b) SUM timings

Figure 11: Experimental results for two block sizes (128, 256) over ClusterData in Upscaledb.

mentation that supports this case where the deletion of a key may increase the storage.

We get the best performance for SIMD codecs (BP128 and SIMD FOR). Unlike other codecs, they show an ability to noticeably improve query performance in all our tests (from small to large databases) on the analytic (SUM) benchmark. Naturally, these gains come with reduced storage usage. As we expected, there is a downside to compression: slower insertion operations. However, for analytic applications where insertions are infrequent, this downside may be inconsequential. The choice between BP128 and SIMD FOR is a trade-off between superior compression (BP128) and superior random lookup speed (SIMD FOR). Indeed, SIMD FOR has superior lookup performance because it supports a binary search directly on the compressed data. However, BP128 has superior compression due to its reliance on differential coding. Our experiments show that BP128 has better performance for low selectivity queries. Our results also show that if we are to use the standard VByte format, then a SIMD-accelerated decoder (Masked VByte) can accelerate queries without requiring any change to the database format.

Our results suggest also that it is beneficial to program common analytic functions (e.g., SUM, COUNT, COUNT DISTINCT, AVERAGE) so that they work directly on buffered data in CPU cache, bypassing explicit cursor handling. Further work could quantify the benefits of implementing these functions so that they operate directly on compressed data.

The importance of SIMD instructions for performance is likely to grow. Already, some processors support wider registers (e.g., 256 bits for recent Intel processors using AVX2 and 512 bits for upcoming processors using AVX-512). From an engineering perspective, it seems easier to design processors that operate on more values during each cycle (wider processors) than processors that run at a higher frequency. Thus it is probably wise to invest in data structures that are best able to benefit from SIMD instructions.

The SIMD accelerated BP128 also offers the best compression ratio, especially for dense key ranges like auto-incremented primary keys or dense time stamps. BP128 compresses data by an order of magnitude, reducing pressure on memory resources.

We limited our work to the compression of 32-bit integer keys. This is a common case well worth optimizing. However, many of the other compression techniques developed for B+-trees could be revisited in light of the new hardware capabilities.

7. Acknowledgments

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Appendix A. Insertions and Deletions in B+-trees

B+-tree [10] can be considered textbook material. Nevertheless, for completeness, we briefly review insertions and deletions in B+-trees. Non-root, non-leaf nodes can accommodate between b and $2b$ keys whereas leaf nodes accommodate between b and $2b - 1$ keys. Nodes are split or merged to maintain the number of keys in these ranges.

Insertion in a B+-tree works generally as a two-step process: we first go down the tree to find the right node, and then we potentially split the nodes from the bottom up. That is, when inserting, we go down the tree to find the leaf node where the insertion is to happen. If the leaf can accommodate another key (its cardinality is less than $2b-1$), it is inserted and the process terminates. When the node is already full, it is split: we divide the node into two nodes containing b keys. One of the new nodes contains the smallest b keys whereas the other one contains the largest b keys. The smallest value of the latter node is copied and added to the parent node as a separator. If needed, the parent node is split too, and the process recurses, possibly all the way to the root. If the root needs to be split, then the height of the tree is effectively increased by one.

Deletion proceeds similarly at first. We find the appropriate leaf node where the key resides. If deleting the key would leave at least b keys in the node, we proceed and the process terminates. If there are two few values, we can examine a neighboring leaf node, having the same parent if possible. If it has more than b keys, it suffices to borrow one of the keys. If taken together, the two leaf nodes have less than $2b$ keys, they need to be merged. Their merger implies at least the removal of the separator key in the parent node, and the process may recurse up to the root, possibly decreasing the height of the tree.