Bachelier: not the forgotten forerunner he has been depicted as

An analysis of the dissemination of Louis Bachelier's work in economics

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Abstract:

This article presents the results of new research on the history of financial economics by analyzing the dissemination of Louis Bachelier's work.

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Louis Bachelier is doubtless the best known French mathematician in the history of modern finance theory. While recent studies have given us a fairly complete picture of the man himself, his work and the results he arrived at, knowledge of his contribution to the development of ideas remains imprecise. Although the direct influence of his work is analyzed on occasion, no study has assessed the dissemination of Bachelier's work, and hence its impact on all scientific disciplines. This is precisely the purpose of this article: to examine the dissemination of Bachelier's work in order to better assess his impact on the development of financial economics[1]. Based on a bibliometric analysis of Bachelier's work, this study aims at shedding light on his influence and explaining how the idea of his "rediscovery" in the 1950s gained credence.

This article demonstrates that, contrary to the widely accepted view, Bachelier's work has never been forgotten; it also shows that the discovery of Bachelier's work by economists has had no significant influence on the development of financial economics.

[1] Jovanovic (2010) makes a similar analysis of the dissemination of Bachelier's work in mathematics.

Louis Bachelier is doubtless the best known French mathematician in the history of modern finance theory. At university he studied mathematics, mechanics and mathematical physics. Although all his work explored the calculation of probabilities and its applications (see the appendix at the end of this article)[2], it is certainly best known for the application to stock exchange operations that he proposed as early as 1900 in his doctoral thesis in mathematical sciences.

Louis Bachelier is generally considered as a formidable forerunner who was forgotten until the mid-1950s. His "rediscovery" is attributed to the American mathematician Leonard Jimmie Savage who, on coming across Bachelier's work published in 1914, sent a postcard to his economist colleagues[3]. Recent work on the history of financial economics has brought Louis Bachelier's discoveries into better focus. It is accepted that Bachelier's thesis is the first known work of mathematics applied to finance (Courtault*, et al.* 2000, Jovanovic 2000, Taqqu

[2] Bachelier defended his thesis in mathematical physics. His research program dealt with mathematics alone: his aim was to construct a general, unified theory of the calculation of probabilities exclusively on the basis of continuous time. However, the genesis of his program of mathematical research most certainly lay in Bachelier's interest in financial markets (Bachelier 1912, 293; Taqqu 2001, 4-5). It seems clear that stock markets fascinated him, and his endeavor to understand them was what stimulated him to develop an extension of probability theory, an extension that ultimately turned out to have other applications.

[3] Bernstein (1992), Walter (1996, 2002), Merton (1998), Scholes (1998), Dimson and Mussavian (1999, 2000), or Whelan, Bowie, and Hibbert (2002).

2001, Davis and Etheridge 2006, Ben-El-Mechaiekh and Dimand 2008)[4]. Also accepted is the fact that, in developing his *Théorie de la spéculation*, Bachelier had at his disposal work published during the 19th century, and although he cites no author in his thesis apart from one mathematician, several clues suggest that he drew directly on the graphical representations of Henri Lefèvre and on Jules Regnault's random walk model[5]. We are also starting to build a better picture of the main writers who were directly or indirectly influenced by Bachelier and thereby gaining a better grasp of the importance of his work and some of his contributions (Taqqu 2001, Davis and Etheridge 2006).

Despite these advances, the fact remains that Bachelier's contribution to the development of scientific ideas has still not been accurately assessed. The main reason for this is that the dissemination of Bachelier's work has not been clearly established. While recent studies have given us a fairly complete picture of the man himself, his work and the results he arrived at, knowledge of his contribution to the development of ideas remains imprecise. Although the direct influence of his work is analyzed on occasion (such and such an author was influenced by Bachelier, or such and such an idea draws on Bachelier's work) no study has

[4] One often hears references to "modern financial theory", but here I am distinguishing between financial economics, meaning economics apply to finance, and financial mathematics, which denotes mathematics applied to finance. This distinction is useful in understanding Bachelier's contribution to the history of science.

[5] See Carraro and Crépel (2006), Jovanovic (2000, 2001a, 2002b), Jovanovic and Le Gall (2001a), Preda (2004), and Taqqu (2001).

assessed the dissemination of Bachelier's work. This is precisely the purpose of this article: to examine the dissemination of Bachelier's work in order to better assess his impact on the development of financial economics. Based on a bibliometric analysis of Bachelier's work, this study aims at shedding light on his influence and explaining how the idea of his "rediscovery" by economists in the 1950s gained credence.

This article is based on a quantitative study that takes a bibliometric analysis as its starting point. The data used were taken from the *Web of Science* and were supplemented by qualitative research based on, among other sources, the *Jstor* online article database. The period extends from 1900 to 2005, and my analysis is based on 440 data. Two points should be borne in mind with regard to the data used.

First, it should be noted that among the references taken from *Jstor*, six references cite Bachelier or mention his name in the body of the text without referring explicitly to a particular piece of writing. In some of these cases, Bachelier's results were mentioned. We attributed these references to the paper by Bachelier that, after reading the articles concerned, seemed the most obvious candidate (*Calcul des probabilités*)[6].

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[6] These were articles by (Dodd 1919), (Doob 1949), (Knibbs 1920), (Melbourne 1925), (Rietz 1923) and an anonymous note published in 1922.

Second, it must be borne in mind that use of the databases of both *Web of Science* and *Jstor* involves a number of biases. Most importantly, since all these databases favour North American journals, North American writers are overrepresented in our database[7]. Consequently, our analysis of the dissemination of Bachelier's work is essentially that of its dissemination in North American journals. Next, the *Web of Science* databases are three in number. They cover about 9,500 journals, but all do not begin at the same period: the *Science Citation Index* goes back to 1900, the *Social Sciences Citation Index* to 1956 and the *Arts and Humanities Citation Index* to 1975. This means there are breaks. To minimize the effects of these breaks on our analysis, I supplemented the data obtained from *Web of Science* with searches in *Jstor*.

This article is divided into two parts.

The first part provides an overview of the dissemination of Bachelier's work between 1900 and 2005. It shows that several periods in the dissemination of his work can be identified, with a marked break at the end of the 1950s (specifically between 1959 and 1961).

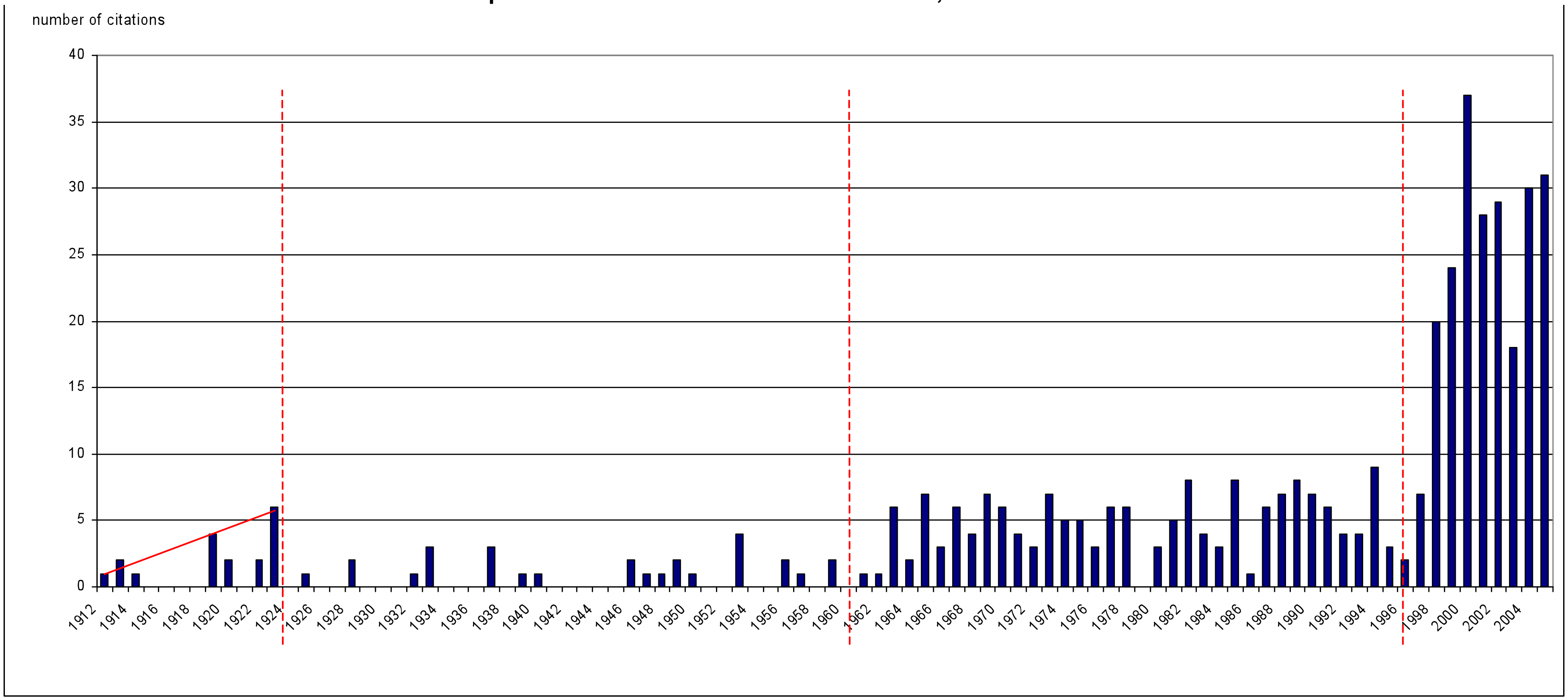
[7] I did not use the databases published by Elsevier in the present work. Although they have the advantage of including European journals, the data are too recent and do not cover the humanities.

The second part of the study analyzes the dissemination of Bachelier’s work in economics. This analysis provides an explanation of the causes of the break at the end of the 1950s.

I. Dissemination of Bachelier's work since 1900

Contrary to what we have thought, Bachelier's work has never been forgotten; on the contrary, as the following graph shows, dissemination of his work began in 1912, the year his *Calcul des probabilités* was published, and has not ceased since.

Graph 1: Dissemination of Bachelier's work, 1990 – 2005



This first graph reveals an interesting point: Bachelier's work was cited during his lifetime[8]. Indeed he rectified errors made in presenting his results in correspondence published in 1913 in *The Mathematical Gazette* (journal of The Mathematical Association).

Graph 1 also allows us to distinguish four periods in the use of Bachelier's work, which have been indicated on the graph:

1912 – 1923

1924 – 1960

1961 – 1997

1998 – 2005

The first period (1912 – 1923) is marked by a growing dissemination of Bachelier's work. The impact of World War I, which created difficulties for publishing in scientific journals, can be clearly seen in the break between 1914 and 1918.

The second period (1924 – 1960) exhibits a discontinuous and relatively weak dissemination of Bachelier's work, with an average of 0.78 citations per year.

The third period (1961 – 1997) is marked by a renewed interest in Bachelier's work, cited without interruption and more frequently (with an average of 4.91

[8] Bachelier died in April 1946.

citations per year). It will be noted that Bachelier's work was cited infrequently between 1961 and 1963, but much more often from 1963 on. The highlight of this period is the publication in 1964 of Paul Cootner's *The Random Character of Stock Market Prices,* in which Bachelier's thesis was translated into English for the first time. This translation facilitated dissemination of Bachelier's work among academics in North America.

The fourth and final period (1998 – 2005) is marked by continuous referencing and an explosion in the number of citations of Bachelier's publications (annual average of 31 citations).

Three major events explain the very widespread dissemination of Bachelier's work in this final period (1998 – 2005).

The first was the award in 1997 of the Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel to Merton and Scholes for their work on options pricing. In their acceptance speech, both men explicitly traced the origin of work in modern financial theory back to Louis Bachelier's thesis, reiterating the broad lines of the rational reconstruction of the history of financial economics from the 1960s.

The second event was the celebration in 2000 of the centenary of the publication of Bachelier's thesis, which was marked by specific publications on his work, the creation of seminars bearing his name, Louis Bachelier learned societies, and websites dedicated to his work.

The third event was the emergence and development, beginning in the mid-1990s, of studies on the history of financial economics that have contributed to the recognition of Bachelier's work.

The evolution of the number of citations of Bachelier since 1900 shown in the first graph hides a huge disparity: as the following table shows, Bachelier's various works have not been cited with the same frequency and have not therefore achieved equal dissemination.

Table 1: respective share of Bachelier's works cited in relation to total citations between entre 1900 and 2005

Publication by Bachelier	Publication date	percentage
Théorie de la spéculation	1900	84,55
Calcul des probabilités	1912	10,23
Le jeu, la chance et le hasard	1914	2,50
Lois des grands nombres	1937	0,91
Proba. à plusieurs variables	1910	0,68
Théorie mathématique du jeu	1901	0,68
Comptes rendus ac. des sciences	1941	0,23
Nouvelles méthodes du calcul des proba.	1939	0,23
Total		100,00

The most frequently cited publications are his thesis, *Théorie de la spéculation*, published in 1900, and his 1912 work, *Calcul des probabilités*[9]. These two publications alone account for 95% of citations, with Bachelier's other works going almost unnoticed.

Let us present briefly these two publications[10].

Théorie de la spéculation, which was also his doctoral thesis, was his first publication. It was the first step of his research program (to construct a general, unified theory of the calculation of probabilities exclusively on the basis of continuous time)[11] and it introduced continuous time probabilities by demonstrating the equivalence between the results obtained in discrete time and in continuous time. In the second part of his thesis he proved the usefulness of this equivalence through empirical investigations of stock market prices.

Because Bachelier's first step in the construction of his general theory of probability calculation was the move from discrete time to continuous time that he demonstrated in his thesis, we understand the key role of his thesis, which he presented in the following manner:

> "The theory of speculation has mainly been useful from the point of view

[9] A comparison of the cited works with the bibliography of Bachelier supplied as an appendix demonstrates just how few of Bachelier's works are cited by North American writers.

[10] Jovanovic (2010) presents Bachelier's scientific aim and his most important publications.

[11] See (Courtault*, et al.* 2002) and Jovanovic (2000).

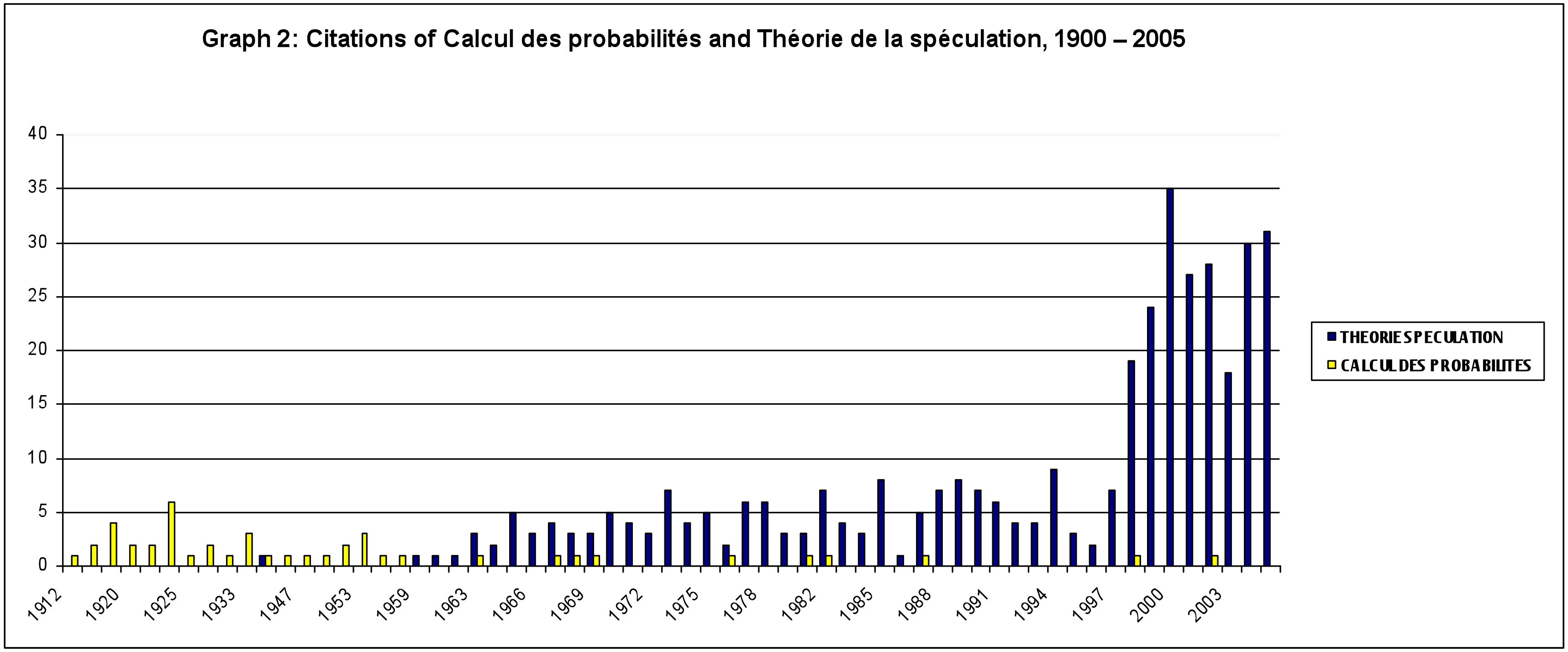
> of pure science; it necessarily introduced into the calculation of probabilities the notion of time and absolute continuity; it has given rise to the theory of continuous probabilities [...]. If speculation did not exist, we would have to invent it" (Bachelier 1914, 177-8).

In 1912, Bachelier published *Calcul des probabilités*. It was through this book that mathematicians learned of Bachelier's work (Jovanovic 2010). The object of *Calcul des probabilités* was to "make known new methods and new results that represent, from certain points of view, a complete transformation of [the calculation of probabilities]. The basis of these new studies is the conception of continuous probabilities […]" (Bachelier 1912, III). The book was based on Bachelier's notes for lectures that he gave at the University of Paris between 1909 and 1914 (Taqqu 2001, 17)[12]. It synthesized and generalized the first results Bachelier had obtained. It should be noted that five of the 23 chapters in the book are devoted to the results of his thesis. More precisely, this book countains a complete presentation of Bachelier's Theory of speculation.

Throughout the period studied, Bachelier's thesis is by far the most frequently cited of his publications (84.5% of total citations). However, this should not obscure the fact that, as graph 1 shows, Bachelier only began to be cited from 1912 onwards – 12 years after the publication of his thesis. This is no coincidence: 1912 was a particularly important year because it saw the

[12] The subject of his courses was "Probability calculus with applications to financial operations and analogies with certain questions from physics".

publication of Bachelier's *Calcul des probabilités.* This work is the publication of Bachelier's that most contributed to the advancement of scientific knowledge (Jovanovic 2010). Graph 2 illustrates this finding since, at the start, only *Calcul des probabilités* is cited, while the doctoral thesis was ignored for close to 60 years.


Graph 2: Citations of Calcul des probabilités and Théorie de la spéculation, 1900 – 2005
40
35
30
25
20
15
10
5
0
1912
1920
1925
1933
1947
1953
1959
1963
1966
1969
1972
1975
1978
1982
1985
1988
1991
1994
1997
2000
2003
THEORIE SPECULATION
CALCUL DES PROBABILITES

Graph 2 shows that Bachelier was first known for *Calcul des probabilités*, and that his thesis began to be cited only in 1959, after which point *Calcul des probabilités* was barely cited at all[13]. Looking only at Bachelier's two main publications, then, two very distinct periods in the dissemination of his work can be discerned:

- 1912 to 1959, when only *Calcul des probabilités* was cited;
- 1959 onwards, when the thesis has been almost the sole publication cited.

These two periods coincide with the four periods observed earlier, because the break at the end of the 1950s is apparent here also. Let us now look more closely at this break.

[13] Except for a single citation in 1937.

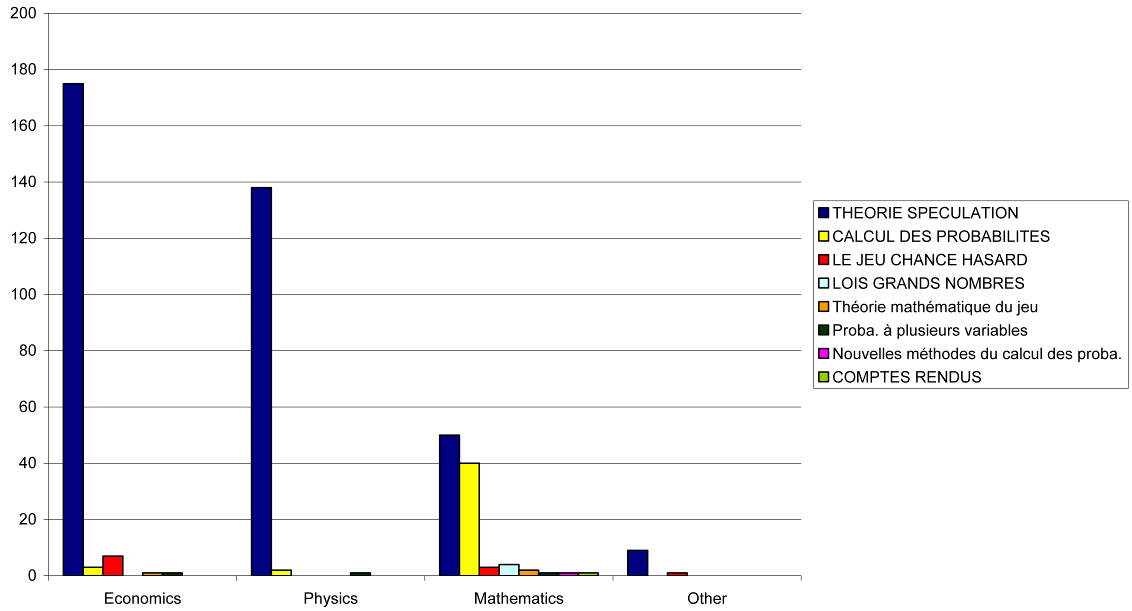
II. Bachelier's work and financial economics

This second section analyzes the manner in which Bachelier's works have been cited by economists. This section seeks to explain the break in the dissemination of Bachelier's work in the 1960s: the time when Bachelier's work began to cited continuously, and with increasing frequency, and when *Théorie de la spéculation* began to be cited while citations of *Calcul des probabilités* virtually disappeared.

Generally speaking, throughout the entire period, articles published in economics journals cite almost exclusively Bachelier's thesis (graph 3)[14].

[14] Note that articles published in mathematics journals cited the widest range of Bachelier's works, and were also those that cited *Calcul des probabilités* most frequently.

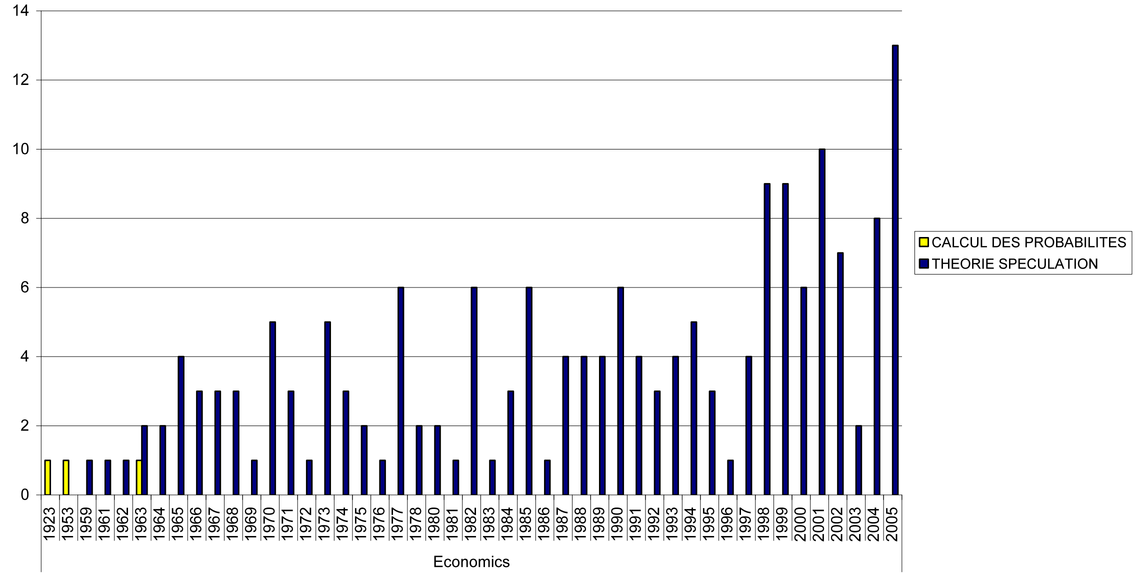
Graph 3: Citations of Bachelier's works by discipline, 1900 – 2005



Furthermore, graph 4 shows that economists only began to cite Bachelier's work from the 1960s onwards, with the exception of two instances, one in 1923 and the other in 1953 – which, moreover, cite *Calcul des probabilités* and not *Théorie de la spéculation*[15]. Lastly, it is only from 1961 onwards that Bachelier's works are cited in economics journals without discontinuity.

Graph 4: Citations of Calcul des probabilités and Théorie de la spéculation in economics journals, 1900 – 2005

[15] These two exceptions are Bowley and Connor (1923) and Allais (1953). While Allais mentioned Bachelier in his references but not in the text, Bowley and Connor used Bachelier for their demonstration (the move from discrete time to continuous time).
Note that Allais (1951) also cited Bachelier.



Two questions arise with regard to the dissemination of Bachelier’s work among economists. First, what explains this belated interest in Bachelier’s work by economists? Second, knowing that Jimmie Savage, a mathematician at Chicago University, is considered responsible for the discovery of Bachelier’s work by economists in the 1960s, what impact did Savage have in economists’ discovery of Bachelier?

I have already shown that it cannot be asserted that Bachelier’s work had remained unknown, since *Calcul des probabilités* was cited from 1912. And yet, one might assume that, because citations of Bachelier’s thesis did not appear until the late 1950s, the applications of Bachelier’s work to financial markets were unknown. Again, this is not the case, since *Calcul des probabilités* re-presents all the results contained in the thesis. Also, the absence of citations of Bachelier’s thesis does not imply ignorance of the possible applications of his work to financial markets. Moreover, it was mathematicians, such as Savage, who drew the attention of economists to this application of the developments of probability theory. As explained elsewhere[16], before modern probability theory had been sufficiently developed in the 1950s, Bachelier's work was used by mathematicians because it was at the leading edge in its field and thus constituted a vital reference[17]. However, Savage was not the first to have brought the usefulness of Bachelier's work for the study of financial markets to the attention of economists: Bachelier's work was applied to analyze financial markets as early as the 1920s.

In December 1922 a session on mathematical statistics was held at the seventh annual meeting of the Mathematical Association of America. Arne Fisher[18] presented a mathematical formula introduced by Bachelier, explaining that:

> "The Bachelier and Gram methods might, for instance, be used to solve the following problem: What is the probability that a certain stock or bond

[16] See Jovanovic (2010) for an analysing of the dissemination of Louis Bachelier's work in mathematics.

[17] Bachelier's works were cited by the period's main contributors to modern probability theory and are often associated with some of the greatest probability theorists of the time, underlining the fact that Bachelier's work was considered sufficiently important and innovative by mathematicians at the time. See, for example, Arne Fisher (1922, x) or Rietz (1923, 155).

[18] In 1915 Arne Fisher, who had immigrated to the United States from Denmark, published an influential book on *The Mathematical Theory of Probabilities and Its Application to Frequency Curves and Statistical Methods* (Shafer and Vovk 2005, 6).

will be quoted at a price x at time t on the stock exchange ?" (in Cairns 1923, 97).

Fisher also showed

> "an actual application he himself had made in the matter of forecasting three months in advance the weekly quotations of a certain gilt-edge stock on the Copenhagen Stock Exchange. During the year of 1922 the lowest value of this stock had been 196 and the highest value 243. The greatest difference between any weekly forecast and the prices actually quoted had been 4 per cent for one of the first weeks of March" (in Cairns 1923, 97).

Fisher used this result to criticize "the investigations by various economists of the so-called business cycles as being the work of mathematical dilettantes" (in Cairns 1923, 97).

Arne Fisher's call was not followed up. But this possible application of Bachelier's work was known, as confirmed by Samuelson, who said that he remembered hearing talk of Bachelier's work as early as the 1930s (Taqqu 2001, 26)[19]. This

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[19] Among mathematicians outside North America who cited Bachelier's work and its application for the study of financial markets were Robert Montessus de Ballore (1870 – 1937) Marcel Boll (1886 – 1971). Montessus de Ballore was a French Professor of mathematics. In his *Leçons élémentaires sur le calcul des probabilités* published in 1908, he wrote a chapter about "speculation" based on Bachelier (1900) in which he called the hypothesis that a speculator's mathematical expectation is zero as "Bachelier's Theorem".

Marcel Boll was a French Professor of physics who ascribes to Bachelier the "fair game theory and speculation (1912)" (1936, 356).

means that the absence of references by economists to Bachelier's work prior to the 1950s cannot be explained by ignorance of its possible application to financial markets[20]. The problem lies elsewhere, and must be sought by looking at the development of modern probability theory.

The history of financial economics is closely linked with the history of modern probability theory (Davis and Etheridge 2006, Jovanovic 2008), to which it owes its major results, hypotheses and models. Let me remind that modern probability theory was properly created in the 1930s, in particular through the work of Kolmogorov, who proposed its main founding concepts (Von Plato 1994). Between the end of the 19th century and the 1930s, the only work being carried out in this new field was the particularly innovative work of mathematicians and physicists. Bachelier was one of these mathematicians. But it was not until after World War II that the Kolmogorov's axioms became the dominant paradigm in this discipline (Shafer and Vovk 2005, 54-5). It is also after World War II that the American probability school was born in the United States. It was led by Doob

[20] We can also mention that Keynes knew Bachelier's Calcul des probabilités and consequently the chapters on speculation and financial markets. However, the two publications in which he cited Bachelier (his 1912 review of Bachelier's *Calcul des probabilités* and in his *Treatise of probability* published in 1921), he never mentioned the applications to financial markets.
We can also note that the *American Economic Review* mentioned in 1914 the publication of Bachelier 1914 book, in which the principles of the theory of speculation is presented.

and by Feller, both of whom cited Bachelier's work very early on[21]. These two writers had a major influence on the construction of modern probability theory, particularly through their two man books published in the early 1950s[22] which proved, on the basis of the framework laid down by Kolmogorov, all results obtained prior to the 1950s, thereby enabling them to be accepted and integrated into the discipline's theoretical corpus. It is also worth noting that after World War II, most American curricula included probability calculus, which greatly contributed to development of the discipline in the United States. In other words, it was only from the 1950s onwards that nonspecialists, and hence economists, began using the tools of modern probability theory (Jovanovic 2010).

As explained elsewhere[23], economists were unable to read the new mathematics developed in Bachelier's doctoral thesis until the 1960s[24]. Consequently, the

[21] Doob explained that he "started studying probability in 1934, and found references to Bachelier in French texts […] The ideas of Bachelier […] made a permanent impression on me, and influenced my work on gambling systems and later on martingale theory" (in Davis and Etheridge 2006, 92).

[22] Doob "finally provided the definitive treatment of stochastic processes within the measure-theoretic framework, in his *Stochastic Processes* (1953)" (Shafer and Vovk 2005, 60). Doob worked on martingale theory from 1940 to 1950. Knowledge of martingale theory was spread gradually during the 1950s, mostly through *Stochastic Processes* (Meyer 2009). This book "became the Bible of the new probability" (Meyer 2009, 3).

[23] See Davis and Etheridge (2006), Jovanovic (2002a).

[24] For instance, Samuelson (1965b, 1965a), who was the first with Mandelbrot (1966) to substitute the martingale model for the random walk model/Brownian

application of continuous time probabilities to financial markets could not be performed by economists[25]. This situation contributes to explain that economists ignored the applications of Bachelier's work for the study of financial markets and that even economists who cited Bachelier's work on speculation before the 1960s did not mention is mathematical results and demonstrations[26].

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motion to represent stock price variations, needed the help of a mathematician to construct his mathematical proof

[25] This difficulty is one of the reasons that explains why financial economics was not constituted as a scientific discipline until the 1960s (Jovanovic 2008).

[26] We know at least two economists who cited the work of Bachelier on speculation before the mid-1950s, Maurice Gherard (1910) and Lucien Laferriere (1951).

Gherardt was a speculator. He used Bachelier for developing a method to speculate on financial markets. He based his analysis only on the statistical results given by Bachelier and by Jules Regnault (1863). However, he completely ignored the mathematical aspects of Bachelier's work.

Lucien Laferriere was professor of Law at the Faculty of Paris. Upon his retirement, July 12, 1951, he offered at the Library of the faculty a set of sheets composing a handwritten book ever published, *La Loi Juridique et la Loi Scientifique de la Bourse* [The legal law and the scientific law of financial markets]. This manuscript was probably the notes of a course addressed to economists. He cited Bachelier but he never used his mathematical demonstration or mathematical results.

For a presentation of Laferriere's manuscript, see Jovanovic (2002a).

For a presentation of Jules Regnault's work, see Jovanovic (2006) or Jovanovic and Le Gall (2001a)

However, note that these publications y are not included in our database (see the introduction about the limits of Web of science).

Knowing this gives us a better picture of Jimmie Savage's "rediscovery" of Bachelier in the mid-1950s. Since Bachelier was already known to American mathematicians (Jovanovic 2010), it is reasonable to assume that Savage, as a mathematician, had been familiar with Bachelier's mathematical work for some time. Why, then, did he send his famous postcard to bring Bachelier to the attention of his economist colleagues? Almost certainly because at the time the potential applications of Bachelier's work to financial markets were ignored to virtually all economists, and few mathematicians had drawn attention to this potential. Savage sent his postcard at a time when some mathematicians were beginning to apply the new mathematics developed in the first half of the 20th century to social sciences[27]. Savage was one of their number and it was his research in mathematics (and more specifically his research into the application of mathematics to social sciences) that led him to look at the application of Bachelier's work to stock market operations[28]. Savage therefore played a role in

[27] I am of course thinking of financial theory (along with modern theory of probability and random processes), but also of game theory, which developed in the second half of the 20th century and saw its first applications in economics after World War II (Leonard 1992, 1995, 2010).

[28] Savage discovered Bachelier's work while translating the work of French mathematician Émile Borel on probability theory: "Three early papers by Emile Borel on minimax solutions to two-person, zero-sum games, originally published from 1921 to 1927, were published in *Econometrica* in 1953, translated into English by Leonard J. Savage with introduction and concluding comment by Maurice Fréchet, the recipient of Lévy's 1943 letter inquiring about Bachelier. Savage's discovery of Bachelier (1914) was thus not quite the isolated fluke Bernstein suggests. Savage was then browsing in the writings of early twentieth-

disseminating Bachelier's work from one discipline to another. It is not surprising, then, that Bachelier's work in finance should be "discovered" by economists from the late 1950s, nor that this discovery came via a mathematician, for whom a reading of Bachelier's work was more accessible.

However, at the time when economists began using stochastic processes and modern probability theory, Bachelier's *Calcul des probabilités* was no longer being referred to by mathematicians, who were now citing only Bachelier's thesis (Jovanovic 2010). Bachelier's results either had been superseded, or had been rewritten in language that integrated Kolmogorov's axiomatic system of probability calculation and subsequent developments. Therefore, people were no longer reading Bachelier, but other mathematicians. A perfect illustration of this point is the case of the mathematician M.F.M. Osborne, who in 1959 published his article on Brownian motion in the stock market; he was unaware of Bachelier's work but referred to more recent results. Furthermore, when the application of Bachelier's work to finance was rediscovered, his mathematical work had lost its innovative character; *Théorie de la spéculation* was at this point cited to provide historical perspective. More particularly, Bachelier would be cited by economists starting from the time that financial economics was created as a

century French probability theorists, and was receptive to the discovery of lost treasures comparable to Borel's contribution to game theory" (Dimand and Ben-El-Mechaiekh 2006, 233). Savage (1972) considered that Borel (1924) review of Keynes' *Treatise of Probability* "contains the earliest account of the modern concept of personal probability known to me".

scientific discipline during the 1960s; he would then be identified by two rational reconstructions of the history of financial economics during the 1960s[29] as the father of the discipline and his thesis identified as the starting point in its history (Jovanovic 2008).

Conclusion

Three main conclusions emerge from this study.

First, contrary to the widely accepted view, Bachelier's work has never been forgotten: mathematicians and economists knew his work since 1912.

Second, Bachelier's work contributed directly to the development of mathematical models and theories until the 1950s[30]. Mathematics is central in the

[29] The inauguration of financial economics as a science and the organization of research in the subdiscipline were accomplished through a particular manner of presenting the history of the discipline. This manner of presentation comes from the construction of the canon of theoretical articles that became the basis of a rational reconstruction of the history. There were two rational reconstructions of the history of financial economics that were created to support the two major theoretical approaches that existed during the 1960s, the first from MIT and the second from the University of Chicago – see Jovanovic (2008).

[30] As Jovanovic (2010) show, mathematicians only began to cite Bachelier's thesis when Bachelier's mathematical work was no longer influencing research work in this field.

Jovanovic (2010) gives a largest analysis on that point.

dissemination of Bachelier's work, which had an impact on the development of knowledge in this discipline only.

Third, the discovery of Bachelier's work – and particularly of his doctoral thesis – by economists provided not so much an analytical support as a kind of handy "off-the-shelf" historical ancestry for the nascent field of modern finance. Indeed, economists discovered Bachelier's work when modern probability theory had been sufficiently developed and mathematicians drew on this new work and no longer on Bachelier's results[31]. I also illustrate the fact that application to stock exchange fluctuations of the mathematics that Bachelier developed could not have been envisaged until the 1960s – a period that saw both the creation of financial economics as a discipline and the development and acceptance of the rational reconstruction of the history of financial economics that propounded an idyllic story of the discovery and dissemination of Bachelier's work.

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[31] Throughout the period in which modern probability theory emerged and developed – from the turn of the 20th century through to the 1930s – *Calcul des probabilités,* the sole publication of Bachelier to be cited, was used by mathematicians. Bachelier's work constituted a vital reference (which explains why Bachelier's name is mentioned along with those of other great mathematicians). During the 1940s and 1950s, mathematicians rigorously proved the main results obtained by Bachelier, thereby making modern probability theory more accessible. Then, his *Calcul des probabilités* ceased being cited and mathematicians looked for the first publication by Bachelier (his thesis) to deal with continuous time probabilities, independently of this first publication's influence.

Before that date, while some economists knew Bachelier's work and its applications for the study of financial markets, they were not interested by them. This point is completely supported by the history of financial economics, which was created during the 1960s. Indeed, before this decade, professors of finance and economists did not use modern probability theory for studying stock markets (Whitley 1986, Jovanovic 2008, Jovanovic and Schinckus 2010)[32].

We can however assert that Bachelier's work was known and appreciated, even if he himself had to fight for recognition of his efforts[33]. Among those outside North America who cited Bachelier's work before the 1960s and that I did not

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[32] This point is confirmed by a remark by Friedman during Markowitz's Ph.D. defence: "This isn't a dissertation in economics, and we can't give you a Ph.D. in economics for a dissertation that's not economics. It's not math, it's not economics, it's not even business administration."

[33] We know the story of the "error" that Paul Lévy believed he had found in Bachelier's work, leading Bachelier to write Lévy to force him to acknowledge his mistake (Taqqu 2001, Courtault and Kabanov 2002). This was not the only incident, as witnessed by the belated, forced recognition by Paul Lévy of another of Bachelier's publications during a lecture on "integrals whose elements are independent random variables" to the *Société Mathématique de France* on April 25,1934:

"Regarding the toss of a coin, Mr. Paul Lévy, having published a dissertation on the subject in 1931, acknowledged the claim of priority of Mr. Bachelier, who in 1912 had published some formulas contained in the dissertation in question, and apologized for not having known about Mr. Bachelier's priority at the time" ("Comptes rendus des séances de l'année 1934", *Bulletin de la Société Mathématique de France* 62: 40-1).

mention yet[34] were also Lucien March (1912, 1930)[35], Louis Gustave du Pasquier (1926)[36], Bohuslav Hostinsky (1932)[37], Paul Lévy (1932, 1934, 1939, 1940)[38], Pierre Delaporte (1944)[39], Robert Fortet (1949)[40], or Corrado Gini (1955)[41].

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[34] Let me precise that they are not included in our database (see the introduction for the limits of Web of science).

[35] March set up the *Institut de Statistique* of Université de Paris. In April 1912, with Alfred Barriol, he invited Bachelier to become a member of the *Societé de statistique de Paris*. Further, the *Journal de la Société de Statistique de Paris* published an obituary of Bachelier (vol. 87, n°5-6, May-June, 1946, p. 7).
For March's work, see Jovanovic and Le Gall (2001b).

[36] Louis-Gustave Du Pasquier (1876 – 1957) was Professor of Mathematics at the University of Neuchâtel. He took his degrees in mathematics in Zürich, but followed courses in the social sciences as well when he spent the year 1900–1901 in Paris at a variety of academic institutions. This book was his textbook of probability (Cramer 2004).

[37] Bohuslav Hostinský (1884 – 1951) was a Professor of Science specialization in Theoretical Physics.

[38] Paul Lévy (1886 – 1971) was a French mathematician specialized in probability theory.

[39] Pierre Delaporte was Professor of Mathematical Statistics.

[40] Robert Fortet (1912 – 1998) was a French mathematician who studied stochastic processes.

[41] Corrado Gini (1884 – 1965) was an Italian statistician, demographer and sociologist who developed the Gini coefficient, a measure of the income inequality in a society.

Appendix: Bibliography of Bachelier

Thesis

Bachelier, Louis. 1900. *Théorie de la Spéculation* [Theory of Speculation], thèse de doctorat ès sciences mathématiques, Université de la Sorbonne, France.

Bachelier, Louis. 1900. *Résistance d'une masse liquide indéfinie pourvue de frottements intérieurs, régis par les formules de Navier, aux petits mouvements variés de translation d'une sphère solide, immergée dans cette masse et adhérente à la couche fluide qui la touche* [*Resistance of an indefinite liquid mass with internal frictions, described by the formulae of Navier, to small translational motions of a solid sphere, submerged in the liquid and adhering to it*], deuxième thèse de doctorat ès sciences mathématiques, Université de la Sorbonne, France.

Books

Bachelier, Louis. 1912. *Calcul des probabilités* (Tome 1), Paris : Gauthier-Villars.

Bachelier, Louis. 1914. *Le Jeu, la Chance et le Hasard*, Paris : Bibliothèque de Philosophie scientifique, Flammarion.

Bachelier, Louis. 1937. *Les lois des grands nombres du Calcul des Probabilités*, Paris : Gauthier-Villars.

Bachelier, Louis. 1938. *La spéculation et le Calcul des Probabilités*, Paris : Gauthier-Villars.

(English Translation: Ben-El-Mechaiekh, Hichem and Robert W. Dimand 2008. *Speculation and the Calculus of Probability*, Working paper)

Bachelier, Louis. 1939. *Les nouvelles méthodes du Calcul des Probabilités*, 1939, Paris : Gauthier-Villars.

Articles

Bachelier, Louis. 1900. « Théorie de la Spéculation », *Annales Scientifiques de l'École Normale Supérieure* : 21-86.

(English Translation;- Cootner (ed.). 1964. *Random Character of Stock Market Prices*, Massachusetts Institute of Technology pp17-78; Davis, Mark and Alison Etheridge. 2006. *Louis Bachelier's Theory of speculation*. Princeton and Oxford: Princeton university press.)

Bachelier, Louis. 1901. « Théorie mathématique du jeu », *Annales Scientifiques de l'école Normale Supérieure* : 143-210.

Bachelier, Louis. 1906. « Théorie des probabilités continues », *Journal de Mathématiques Pures et Appliquées* : 259-327.

Bachelier, Louis. 1908. « Étude sur les probabilités des causes », *Journal de Mathématiques Pures et Appliquées* : 395-425.

Bachelier, Louis. 1908. « Le problème général des probabilités dans les épreuves répétées », *Comptes-rendus des Séances de l'Académie des Sciences*, Séance du 25 mai : 1085-1088.

Bachelier, Louis. 1910. « Les probabilités à plusieurs variables », *Annales Scientifiques de l'école Normale Supérieure* : 339-360.

Bachelier, Louis. 1910. « Mouvement d'un point ou d'un système matériel soumis à l'action de forces dépendant du hasard », *Comptes-rendus des Séances de l'Académie des Sciences*, Séance du 14 novembre, présentée par M. H. Poincaré : 852-855.

Bachelier, Louis. 1913. « Les probabilités cinématiques et dynamiques », *Annales Scientifiques de l'École Normale Supérieure* : 77-119.

Bachelier, Louis. 1913. « Les probabilités semi-uniformes », *Comptes-rendus des Séances de l'Académie des Sciences*, Séance du 20 janvier, présentée par M. Appell : 203-205.

Bachelier, Louis. 1915. « La périodicité du hasard », *L'Enseignement Mathématique* : 5-11.

Bachelier, Louis. 1920. « Sur la théorie des corrélations », *Comptes-rendus des Séances de la Société Mathématique de France*, Séance du 7 juillet : 42-44.

Bachelier, Louis. 1920. « Sur les décimales du nombre π », *Comptes-rendus des Séances de la Société Mathématique de France*, Séance du 7 juillet : 44-46.

Bachelier, Louis. 1923. « Le problème général de la statistique discontinue », *Comptes-rendus des Séances de l'Académie des Sciences*, Séance du 11 juin, présentée par M. d'Ocagne : 1693-1695.

Bachelier, Louis. 1925. « Quelques curiosités paradoxales du calcul des probabilités », *Revue de Métaphysique et de Moral* : 311-320.

Bachelier, Louis. 1941. « Probabilités des oscillations maxima », *Comptes-rendus des Séances de l'Académie des Sciences*, Séance du 19 mai : 836-838.

References

Allais, Maurice. 1951. Notes théoriques sur l'incertitude de l'avenir et le risque. *d'Econométrie, Congrès Européen*. September.

------------. 1953. Le comportement de l'homme rationnel devant le risque: critiques des postulats et axiomes de l'Ecole américaine. *Econometrica* 21. 4: 503-46.

Anonymous. 1922. Notes. *The American Mathematical Monthly* 29. 7: 266-268.

Bachelier, Louis. 1900. Théorie de la spéculation reproduced in. *Annales de l'Ecole Normale Supérieure, 3ème série* 17. January: 21-86. Reprint, 1995, J. Gabay, Paris.

------------. 1912. *Calcul des probabilités*. Paris: Gauthier-Villars.

------------. 1914. *Le jeu, la chance et le hasard*. Paris: E. Flammarion.

Ben-El-Mechaiekh, Hichem and Robert W. Dimand. 2006. Louis Bachelier. In *Pioneers of Financial Economics, Volume 1: Contributions Prior to Irving Fisher*, edited by Poitras, Geoffrey, 225-237. Cheltenham: Edward Elgar Publishing.

------------. 2008. *Louis Bachelier's 1938 Volume on the Calculus of Speculation: Efficient Markets and Mathematical Finance in Bachelier's Later Work*.Working paper.

Boll, Marcel. 1936. *La Chance et les Jeux de Hasard*. Paris Larousse.

Borel, Émile. 1924. A propos d'un traité de probabilités. *Revue Philosophique* 98. 321-336.

Bowley, Arthur Lyon and L. R. Connor. 1923. Tests of Correspondence between Statistical Grouping and Formulae. *Economica* 7. 1-9.

Cairns, W. D. 1923. Seventh Annual Meeting of the Mathematical Association of America. *American Mathematical Monthly* 30. 3: 87-104.

Carraro, Laurent and Pierre Crepel. 2006. Louis Bachelier 11 mars 1870 - 28 avril 1946. *Images des mathématiques* 2006. 48-49.

Cootner, Paul H. 1964. *The random character of stock market prices*. Cambridge, Mass.: M.I.T. Press.

Courtault, Jean-Michel and Youri Kabanov. 2002. *Louis Bachelier: aux origines de la finance mathématique*. Besançon: Presses Universitaires Franc-Comtoises.

Courtault, Jean-Michel, Youri Kabanov*, et al.* 2000. Louis Bachelier on the century of Théorie de la spéculation. *Mathematical Finance* 10. 3: 341-353.

------------. 2002. Louis Bachelier on the centenary of Théorie de la spéculation. In *Louis Bachelier: aux origines de la finance mathématique*, edited by Courtault, Jean-Michel and Youri Kabanov, 5-86. Besançon: Presses Universitaires Franc-Comtoises.

Cramer, Jan Salomon. 2004. The early origins of the logit model *Studies in History and Philosophy of Biological and Biomedical Sciences* 35. 4: 613-626.

Davis, Mark and Alison Etheridge. 2006. *Louis Bachelier's Theory of speculation*. Princeton and Oxford: Princeton university press.
Delaporte, P. 1944. Les nouvelles méthodes du calcul des probabilités. *Journal de la Société de statistique de Paris* 86. 3-4: 96.
Dodd, Edward. 1919. Reviewed work(s): Vital Statistics, An Introduction to the Science of Demography. by G. C. Whipple. *The American Mathematical Monthly* 26. 10: 452-454.
Doob, Joseph L. 1949. Heuristic Approach to the Kolmogorov-Smirnov Theorems. *Annals of Mathematical Statistics* 20. 3: 393-403.
Fisher, A. C. 1922. *Elementary Treatise on Frequency Curves and their Application to the Construction of Mortality Tables*. New York: The MacMillan company.
Fortet, R. 1949. Quelques travaux récents sur le mouvement brownien. *Annales de l'institut Henri Poincaré* 11. 4: 175-226.
Gherard, Maurice. 1910. *Le gain mathématique à la Bourse*. Paris Charles Amat.
Gini, Corrado. 1955. Sur quelques questions fondamentales de statistique. *Annales de l'institut Henri Poincaré* 14. 4: 245-364.
Hostinský, B. 1932. Application du Calcul des Probabilités à la Théorie du mouvement Brownien. *Annales de l'institut Henri Poincaré* 3. 1: 1-74.
Jovanovic, Franck. 2000. L'origine de la théorie financière: une réévaluation de l'apport de Louis Bachelier. *Revue d'Economie Politique* 110. 3: 395-418.
------------. 2002a. La Loi Juridique et la Loi Scientifique de la Bourse par Julien Laferrière. *Revue d'Histoire des Sciences Humaines* 7. 181-191.
------------. 2002b. Instruments et théorie économiques dans la construction de la "science de la Bourse" d'Henri Lefèvre. *Revue d'Histoire des Sciences Humaines* 7. 41-68.
------------. 2006. A nineteenth-century random walk : Jules Regnault and the origins of scientific financial economics. In *Pioneers of Financial Economics, vol. 1*, edited by Poitras, Geoffrey, 191-222. Cheltenham: Edward Elgar.
------------. 2008. The Construction of the Canonical History of Financial economics. *History of Political Economy* 40. 2: 213-242.
------------. 2010. *The dissemination of Louis Bachelier’s work in mathematics*.Working paper.
Jovanovic, Franck and Philippe Le Gall. 2001a. Does God practice a random walk? The "financial physics" of a 19th century forerunner, Jules Regnault. *European Journal for the History of Economic Thought* 8. 3: 323-362.
------------. 2001b. March to numbers: The statistical style of Lucien March. In *History of Political Economy, annuel supplement: The Age of Economic Measurement*, edited by Klein, Judy and Mary S. Morgan, 86-100.
Jovanovic, Franck and Christophe Schinckus. 2010. *l'émergence de l'économie financière dans les années 1960*.
Keynes, John Maynard. 1912. Review of Louis Bachelier's Calcul des probabilités. *Journal of the Royal Statistical Society* December. Reedited in 1983 in the Collected Writings, Londres : Macmillan and Cambridge University Press, volume XI, pp. 568-73:

------------. 1921. *A Treatise on Probability*. Londres: The MacMillan company.
Knibbs, G. H. 1920. The Analysis of a Census. *Quarterly Publications of the American Statistical Association* 17. 130: 177-187.
Laferrière, Julien, and 1951. La loi juridique et la loi scientifique de la Bourse. Cujas Library (cote : MS 196)
Leonard, Robert J. 1992. Creating a Context for Game Theory. *History of Political Economy* 24. Special Issue: Towards a History of Game Theory: 29-76.
------------. 1995. From Parlor Games to Social Science: Von Neumann, Morgenstern, and the Creation of Game Theory, 1928-1994. *Journal of Economic Literature* 33. 2: 730-761.
------------. 2010. *Von Neumann, Morgenstern, and the Creation of Game Theory From Chess to Social Science, 1900–1960*. Cambridge: Cambridge University Press.
Lévy, Paul. 1932. Sur un problème de calcul des probabilités lié à celui du refroidissement d'une barre homogène. *Annali della Scuola Normale Superiore di Pisa - Classe di Scienze, Sér. 2* 1. 3: 283-296.
------------. 1934. Sur les intégrales dont les éléments sont des variables aléatoires indépendantes. *Annali della Scuola Normale Superiore di Pisa - Classe di Scienze, Sér. 2* 3. 3-4: 337-366.
------------. 1939. Sur certains processus stochastiques homogènes. *Compositio Mathematica* 7. 283-339.
------------. 1940. Le mouvement brownien plan. *American Journal of Mathematics* 62. 487-550.
Mandelbrot, Benoit. 1966. Forecasts of Future Prices, Unbiased Markets, and "Martingale" Models. *Journal of Business* 39. 1, Part 2: 242-255.
March, Lucien. 1912. Calcul des probabilités, par Louis Bachelier. *Journal de la Société de statistique de Paris* 53. 6: 258-9.
------------. 1930. *Les Principes de la Méthode Statistique*. Paris: Félix Alcan.
Melbourne, S. Applegate. 1925. Reviewed work(s): Statistical Methods Applied to Economics and Business. by Frederick Cecil Mills. *Journal of the American Statistical Association* 20. 150: 295-296.
Meyer, Paul-André. 2009. Stochastic Processes from 1950 to the Present. *Journal Electronique d'Histoire des Probabilités et de la Statistique* 5. 1: 1-42.
Montessus de Ballore, Robert (Vicomte de). 1908. *Leçons élémentaires sur le calcul des probabilités*. Paris: Gauthier-Villars.
Pasquier, (du) L.-G. 1926. *Le Calcul des probabilités. Son évolution mathématique et philosophique*. Paris: J. Hermann.
Preda, Alex. 2004. Informative Prices, Rational Investors: The Emergence of the Random Walk Hypothesis and the 19th Century "Science of Financial Investments". *History of Political Economy* 36. 2: 351-386.
Regnault, Jules. 1863. *Calcul des chances et philosophie de la bourse*. Paris: Mallet-Bachelier and Castel.
Rietz, H. L. 1923. On the Subject Matter of a Course in Mathematical Statistics. *The American Mathematical Monthly* 30. 4: 155-166.

Samuelson, Paul A. 1965a. Rational theory of warrant pricing. *Industrial Management Review* 6. 2: 13-40.
------------. 1965b. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6. 2: 41-49.
Savage, Leonard Jimmy. 1972. *The foundations of statistics*. New York: Dover.
Shafer, Glenn and Vladimir Vovk, and 2005. The origins and legacy of Kolmogorov's Grundbegriffe.
Taqqu, Murad S. 2001. Bachelier and his Times: A Conversation with Bernard Bru. *Finance and Stochastics* 5. 1: 3-32.
Von Plato, Jan. 1994. *Creating modern probability: its mathematics, physics, and philosophy in historical perspective*. Cambridge [England]; New York: Cambridge University Press.
Whitley, Richard Drummond. 1986. The Rise of Modern Finance Theory: its characteristics as a scientific field and connection to the changing structure of capital markets. In *Research in the History of Economic Thought and Methodology*, edited by Samuels, Warren J., 147-178. Stanford: JAI Press Inc.